Sirte University Scientific Journal Vol. 14, No. 1 (2024) 30-39

Sirte University Scientific Journal (SUSJ)

Journal home page: http://journal.su.edu.ly/index.php/SUSJ/index

DOI: 10.37375/ susj.v14i1.2798

The Implementation of Fuzzy Logic and Fast Fourier Transformation in Image Processing (Linear & Non-Linear Filters)

Aya Zidan Mohammed[1], Najat Ali Atbaiga[2] Souad Abummaryam[3] and Naiema Mousa Amrayid[4]

[1,3]*Department of Mathematics, Faculty of Science, Sirte University, Libya.*

[2,4] *Department of Computing, Faculty of Information Technology, Sirte University, Libya.*

DOI: /10.37375/susj.v14i1.2798

ARTICLE INFO:

Received 13 February 2024.
Accepted 10 May 2024.
Available online 01 June 2024.

Keywords:

Fuzzy Logic, Spatial Domain, Frequency Domain, Fourier Transform, Image Processing.

A B S T R A C T

Fuzzy logic has several impacts on our daily life, and also offers a solution of non-linear problem of real world; expert systems and artificial intelligence. The use of fuzzy logic provides flexibility which is perfect for development of computer vision and image processing technologies. Image processing is a technique for adding effects to an image, obtaining an improved image, or obtaining some relevant information. The Fourier transform has a wide range of industrial applications, but it makes a significant contribution to image processing domains including image enhancement and restoration. This paper introduces applying the frequency domain (Fourier transform) and Spatial domain in image processing. The work divided to three main sections, the first section views literature review of Fuzzy logic, applications of fuzzy logic .Fast Fourier transform. The second section presents image processing; includes Frequency and spatial domain. Spatial domain proposes some selected filters (linear and non-linear). Frequency domain displays Fast Fourier transform equations to convert the processed data to fuzzy representation. Particularly Fourier theory of image processing procedures implemented using "MATLAB" software. The final section concluded the results and conclusion.

1 Introduction

The **Boolean logic** is extended by the idea of Fuzzy Logic. It is a method of modelling logical reasoning where the truth of the statement is not binary. In stream of continuous representation of data Fuzzy logic represents uncertainty. The best development in several scientific domains; particularly Computer Science and through the use of fuzzy logic is "flexibility". Computer vision applications including photography and printing, face detection, and identification require image processing as a prerequisite; that has clear impact by fuzzy logic. Image processing and computer vision have many application in facial and object recognition. Compared to traditional logical systems, fuzzy logic is much more general. Fuzzy logic presents a foundation for the development of new tools for natural language processing such as computing with words. Mandwe &Anjum (2016).

Fuzzy sets are used to create the Fuzzy logics and; using Fuzzy logics can design Fuzzy experts systems. The technique of image processing can be used to perform some operations on an image, to be enhanced, or extract some useful information. Image processing deals with storing, filtering, and other operations on signals; which is pre-requisite of computer vision applications. Most of image processing techniques involve using standard signal processing techniques and treating the image as two-dimensional signals. Jiang & Chai (2015)

Several transformations such as the Fourier transform and the fuzzy transform; have been proposed to solve problems that range from signal analysis to the solution of partial differential equations. The range of problems is clear from the analysis stage to the approximation stage of signals, as well as from fuzzy logic to fuzzy modeling. The growing of data availability that supported by ongoing technological advancements in acquisition, storage, and processing. Patane (2020). The Fast Fourier transform is a well know method that is widely used in image processing. In general, we can say that the Fast Fourier transform converts a function (image) considered to a frequency domain. Transformation to a frequency domain is a very important tool in many applications. For example, applying filters to an image in a frequency domain is computationally faster than in an image domain. The Fuzzy transform applications developed special tools that are used as methodologies to image processing. Many problems in image processing can be solved by applying various types of transforms that are considered as powerful methods.

This paper proposes the applying of frequency domain (Fourier transform) in image processing. The first section examines the literature review of Fuzzy logic and Fourier transform. The second section shows mathematical equations of some spatial Filters; a verging and Gaussian as linear filters and, Median and Max & Min filter as nonlinear filters. The implementation, discussion and conclusion of the results, are presented in the final sections.

2 Related work

Several approaches have been proposed for mathematical theories that suggested for the use of fuzzy logic, and many papers have looked at fuzzy logic in different ways, some of them are introduced here.

Gómez-Echavarría (2020) indicated that the classical Fourier transform has been proposed to be generalized as fractional Fourier Transform FrFT. The pattern recognition and automatic diagnosis purposes can be achieved by using fractional spectral signatures. Moreover, the fractional order is used to characterize non-stationary dynamics, and providing relevant physiological information by determines time-frequency changes. The implementing FrFT in biomedical solutions considered as a novel data analysis tool which supports its usefulness and Potential use.

Kaur & Sinch (2019) reviewed the basic concept of fuzzy sets, fuzzy logics and fuzzy expert systems. The implementing of fuzzy sets presented optimal solution of any problem of any field, such as robotics, image processing etc; with results were remarkable.

Kanwal et al (2019) presented two approaches for detecting image splicing. First approach uses gray values of the image to extract local binary pattern LBP or local ternary pattern LTP features. Second approach uses Fast Fourier Transform to extract the ELTP features. The Results of applying these techniques observed that the features of LBP and ELTP are found to be more effective in classifying images as forged. The FFT-ELTP technique is discovered to be significant feature compared to other image splicing detection techniques. Complex transformations like FFT are used in presented approaches to increase the complexity of the process.

S. Shi, R. Yang and H(2017) proposed a new algorithm called 2D-SFFT which utilizing of the sparsity features of images. The results of Experimental indicated that 2D-SFFT offers a significant advantage than standard 2D-FFTW, specially with running time for 2D-SFFT has the faster runtime than 2D-FFTW for a wide range of image size.

Souverville et al (2015), introduced algorithms that use fuzzy logic techniques to improve the low resolution of the image. The results have discussed after analyzing different interpolation methods. Losing the image details such as preserving edge and wasting more processing time, come as disadvantages with this techniques. The main point presented as advantages; is an image with more edge defined.

Petric (2015), described algorithm that provide ability to choose specific edges to be displayed. The edges can be classified by the membership function. The disadvantage of this approach is time complexity; where this problem can be solved by the features of chosen programming language. The ease of implementation is mentioned as the main advantages of selected algorithm.

3 The Fuzzy Logic

The Fuzzy Logic is decision-making and deals with ambiguous and imprecise data. This is a gross simplification of real-world problems, and it is based on degrees of truth rather than True/False or 1/0, as in Boolean logic. Fuzzy logic is a method for dealing with uncertainty that combines real numbers [0...1] and logic operations which based on fuzzy set theory and fuzzy set membership, both of which are frequently used in natural(spoken) language. Frank (2013)

3.1 Fuzzy Logic Processes

The basic process of fuzzy logic controller consists of mainly three steps known as Fuzzification, Rule Base Inference engine, and Defuzzification as shown in fig.1. At the first stage, the input and the output of the system is need to be clearly defined.

1. Fuzzification process is used to transformed the crisp input to linguistic variable "fuzzy set" by the membership functions stored in the fuzzy knowledge base.
2. The second step is Rule Base Fuzzy Inference Engine or fuzzy associative memory. In this step fuzzy logic uses if-then type fuzzy rule to converts the fuzzy input to the fuzzy output. This stage is used to assign the decision or control action rules based upon the input values. It is a fuzzy Truth table which shown all possible output for all possible inputs. If there are many rules activated at a time. Then, their aggregation is carried out to obtain the single output. This aggregation is carried out by fuzzy operators. A simple MAX-MIN method of selection is used where the maximum value of the inferences is used as the final conclusion.
3. The last step is Defuzzification process that performed to convert the fuzzy value obtained from composition into a "crisp" value. It is the conversion of fuzzy quantity to a precise quantity, in reverse of Fuzzification. Defuzzification is necessary, since controllers of physical system require discrete signals. Some commonly used defuzzification methods are: Centre of gravity (COG), weight average method, and mean of maximum (MOM) method

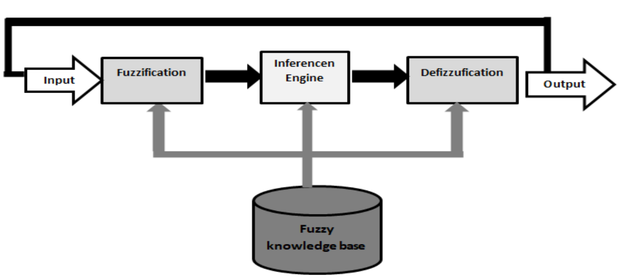

Fig.1 Fuzzy logic processes

3.2 The Fuzzy set

The concept of fuzzy sets is an extension of the notation of the classical set. A fuzzy set differs from a crisp set in that its members may have different degrees of membership depending on how closely meet the membership required in the context of set membership. It essentially permits partial membership, which means it can contain elements with varying degrees of set membership. Fuzzy set generalize this idea by expanding the characteristic function from {0, 1} the entire interval [0, 1]. Petric (2015).

Membership function: A membership is a function in fuzzy set that maps a set of elements to their corresponding grades of truth. The Member function μ_A can be used to define a set X as:

$$\mu_A(x) = \begin{cases} 1, & if \quad x \in A \\ 0, & if \quad x \notin A \end{cases} ; \mu_A : X \rightarrow [0,1]$$

Where X: any set, A: values in the range from 0 to 1.

3.3 The Applications of fuzzy logic

Applications of fuzzy sets and fuzzy logic can befound in many fields, including artificial intelligence, enginee ring,computer science, operations research,robotics, an d pattern recognition. Applications in image processing in these fields have also matured. Lotfi (2013).

Many commercial applications that use fuzzy logic exist today. A few examples are listed here: Timoth (2010).

1. **Securities:** Decision systems for securities trading, various security appliances Transportation: Train schedule control Railway acceleration, Automatic underground train operation, , Braking and stopping.
2. **Electronics:** Control of automatic exposure in video cameras, Air conditioning systems, Washing machine timing, Humidity in a clean room, Microwave ovens, and Vacuum cleaners.
3. **Medical:** Medical diagnostic support system, Control of arterial pressure during anesthesia, Multivariable control of anesthesia, Modeling of neuropath logical findings in Alzheimer patients, Radiology diagnoses, Fuzzy inference diagnosis of diabetes and prostate cancer.
4. **Pattern Recognition and Classification:** Fuzzy logic-based speech recognition

Handwriting recognition, Fuzzy logic-based facial characteristic analysis, Command analysis, Fuzzy image search Psychology: Fuzzy logic-based analysis of human behavior, Criminal investigation, and prevention based on fuzzy logic reasoning.

5. **Fuzzy logic application in Image process:** Fuzzy Image processing is Collection of all approaches that understand, represent and process the images, their segments, and features as fuzzy sets. The representation and processing based on the selected fuzzy technique and on the problem to be solved. Mahashwari & Asthana (2013).

4 Imasge Processing

Image processing isa combination of artificial intelligen ce and computer vision.
These processes convert image into digital form and pe rform certain operations to extract useful information o r improve it, Tonyyan et al (2020). In addition; all images treated as 2D signals in image processing system when pre-determined signal processing methods are applied. The most common task of image processing is enhancement image quality. The analogue and digital image processing are two methods used for image processing. Vinay et al (2020).

An image, as defined in the "real world" is considered to be a function of two real variables, $:R^2 \rightarrow R$; $f_0(x,y) = h(x,y) + u(x,y) + f(x,y)$ **(1)**

Where $f_0(x,y)$ is scene's observe image, $f(x,y)$ is degradation, $h(x,y)$ is the degradation linear operator, $u(x,y)$ is the additive noise. Pratt (2013)

In digital image processing systems,these are usually n umerical fieldsobtained by scanning points in space ofa physical image. After processing, another series of nu mbers is generated, which are used to reconstruct consecutive images for viewing. Computer algorithms must read to perform image processing on digital image. Kumar & Nanda (2008).

The approaches of edge detection have a major part in image processing tasks. The main aim of edge detection is to mention the differences and determine the physical occurrence. The differences in image intensity between one pixel to another, are known as Edge. Edge detection in the noisy image is difficult due to high-frequency contents of both the noise and edge. There are many edge detection methods, available and each developed with a specific features. Edge detection has many ways to be performed, but the most of the techniques can be divided into two types: Bickey et al (2020).

1. Gradient-based edge detection: edge is found by locating the maximum and minimum in the first derivative of the image.
2. Laplacian based edge detection: It searches of zero junctions in the second derivative of the image. The location can be easily defined by calculating the derivative of the image where the edge has the design of a ramp which is one-dimensional.

4.1 The Fundamental Steps

The basic steps of image processing as the following are produced: Vese & Wittman (2010).

1. **Simple image formation model:** An image may be defined as a two-dimensional function $f(x,y)$, the value f at the spatial coordinates (x,y) is positive where determined by the source of the image. Therefore, $f(x,y)$ must be nonzero and finite: $0 < f(x,y) < \infty$. The image-function f may be characterized by two components: $i\,(x,y)$ depends on the illumination source, and $r(x,y)$ depends on the reflectance:

We have $f(x,y) = i(x,y)\,r(x,y)$ **(2)**

Where: $0 < i(x,y) < \infty$ *and* $0 < r(x,y) < 1$

Image sampling and quantization: There are two steps involved in converting the continuously detected data into digital format:

i. Sampling: digitizing the coordinate values.

ii. Quantization: digitizing the amplitude values A digital image is represented as $M \times N$ matrix:

$$f = \begin{bmatrix} f_{0,0} & f_{o,1} \cdots . & f_{0,N-1} \\ f_{1,0} & f_{1,1} \cdots . & f_{1,N-1} \\ f_{M-1,0} & f_{M-1,1} \cdots . & f_{M-1,N-1} \end{bmatrix}$$

The number of gray level; L is taken to be a power of 2, $L = 2^k$ for some integer $K > 0$, many times, digital images take values 0,1,2,….,255, thus 256 distinct gray levels.

2. **Intensity transformations**: performs contrast modification and image thresholding on a single pixel in an image.
3. **Histogram equalization:** This technique is a statistical tool to improve the contrast of images. The input image is $f(x,y)$ (as a low contrast image, dark image, or light image), The output is a high contrasted image $g(x,y)$.There are two ways of performing the two-dimensional image. The first; is to directly process the two-dimensional image in the spatial domain and the second method is to process the image in the frequency domain.

4.2 The Spatial Domain

Spatial domain filtering is a pixel neighborhood operation commonly used spatial filtering techniques, include median filtering, average filtering, Gaussian filtering and etc.. can be broadly classified into two different categories namely, linear filtering and order-statistic filters. And hence the classification as follows:

1. **Linear Spatial Filtering** (convolution): The operation can be convolution. The value of a given pixel in the output image is represented by the weighted sum of the pixel values of its neighborhood (linear combination) in the input image. Linear filtering of an image f of size $M \times N$ with a filter mask of size $m \times n$ is given by the expression: Vese &Wittman (2010)

$$(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \quad \textbf{(3)}$$

$where\ a = \frac{m-1}{2}\ and\ b = \frac{n-1}{2}\ x = 0,1,2,\ldots,M-1\ and\ \ y = 0,1,2,..,N-1.$

i. **Smoothing Spatial Filters** (low pass) filter. includes:

- **Averaging linear filter**: The response of the averaging filter is simplify the average of the pixels contained in the neighborhood of the filter mask. The first one provides the average filter or the box filter, $w = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Is called standard a verge filter of size 3×3. The general weighted a verge filter of size $m \times n$,

$$g(x,y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)} \quad \textbf{(4)}$$

- **Gaussian Filtering**: it uses to fuzzy images and reduce noise and detail. Gaussian kernel coefficients are sampled from the 2D dimensional Gaussian function:

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \quad \textbf{(5)}$$

Where σ is the standard deviation of the distribution that is assumed to have a mean of 0.

ii. **Sharpening Linear Filters** (high pass) filter: presents a method for image enhancement using the Laplacian of F in the continuous case by:

$$\Delta f(x,y) = \frac{\partial^2 f}{\partial x^2}(x,y) + \frac{\partial^2 f}{\partial y^2}(x,y) \quad \textbf{(6)}$$

The equation can be implemented using 3×3 mask $w = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

The Laplacian operator includes the sharpening effect on the original:

$$g(x,y) = f(x,y) + \Delta^2 f(x,y) \quad \textbf{(7)}$$

2. **Order-statistics Nonlinear Filters**: Order-statistics filters are spatial filters. The response of these filters is determined by ordering (ranking) the pixels contained in the image region that encompassed: Kaur & Snich (2019).

i. **Median Filter**: replaces the value at the center with the median pixel value in the neighborhood (i,e the middle element after sorting). Median filters are particularly used to remove impulse noise (also known as salt and pepper noise=0). Salt=255, gray levels. The median of 2N+1 sample is the value which has N smaller or equal values and N larger or equal to it.

ii. **Max and min Filter**: Using the 100^{th} percentile results in so-called max filter, given by: $f(x,y) = max_{(s,t)\in s_{xy}}\{g(s,t)\}$

The brightest points in an image can be determined by applying this filter. Since pepper noise has very low values, the noise can be reduced as a result of the max selection processing of the sub-image area $s_{(x,y)}$.

The 0^{th} percentile filter in min filter:

$$f(x,y) = min_{(s,t)\in s_{xy}}\{g(s,t)\}$$

The darkest point in an image can be found by this filter as result of using the min operation to reduce salt noise.

4.3 The Frequency domain

Arithmetic operations are used to convert an image into a frequency distribution operation in the frequency domain. The processed data transferred back to the cosmic field through the inverse mathematical transformation. Mathematical transformation and frequency domain processing can also be performed using various methods, such as Fast Fourier transform, Hadamard transform, Walsh transform, Hotel transform, and waveform image processing.

1. The Fourier Theory

The Fourier transform has many uses in engineering across the board, but it particularly excels in areas such as image enhancement. Using Fuzzy fast Fourier transform, the image is initially converted to the fuzzy representation where each pixel is represented in terms of degree of membership. Once an image is transferred to the Fast Fourier transform then decomposes the image into two different components; (real and imaginary) which the frequency domain representation of concerned image. The number of frequencies of an image is comparable to the number of pixels in the image or spatial domain. The process of inverse transform is the frequencies that are transformed into the original image. Vani & Hema (2020).

2. Fourier Transform

The composition of an image to its sine and cosine components can be using the significant tool of image processing; Fourier Transform. The image is represented in the frequency domain as output of the transformation while the input image represented in the spatial domain equivalent. Alejandro et al (2020). The equations of original image and its inverse are represented in Fast Fourier transform by the following formula:

$$FFT(x,y) = \sum_{m=0}^{M-1}\sum_{p=0}^{p-1} fun(u,v)e^{-j2\pi\left(x\frac{u}{U}*y\frac{v}{V}\right)}$$

$$InvF(v) = \frac{1}{p}\sum_{p=0}^{p-1} fun(x)e^{j2\pi(x\frac{v}{V})}$$

Where $F(u,v)$ is the pixel at coordinates (u,v), $FFT(x,y)$ is the frequency domain related to the coordinates x and y of the image, and u and v are the image dimensions.

The Fast Fourier transform is used to access the geometric characteristics of a spatial domain image due to the image in the Fourier domain is decomposed into its sinusoidal components. Examining or processing certain frequencies of the image, would be simple, consequently influencing the geometric structure in the spatial domain.

3. Filtering in the Frequency Domain:

It consists of computing the inverse transform after modifying the Fast Fourier transform of an image to obtain the processed results. A lot of digital filters are built in the frequency domain, where input signals are defined by their frequency spectrum and design filters to modify that spectrum. Panchi & Hong (2018).

The equations of basic steps of filtering in the frequency domain are: Vese &Wittman (2010)

- **i.** Compute $F(u,v) = 2DFT(f(x,y)(-1)^{x+y}$, the (DFT) discrete fuzzy transformation of the input image.
- **ii.** Multiply $F(u,v)$ by a filter function:

$$H(u,v) = 1 + 4\pi^2(\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N^2}{2}\right))$$

$$G(u,v) = H(u,v)F(u,v).$$

- **iii.** Compute the inverse DFT of the result
- **iv.** Obtain the real part of the inverse DFT

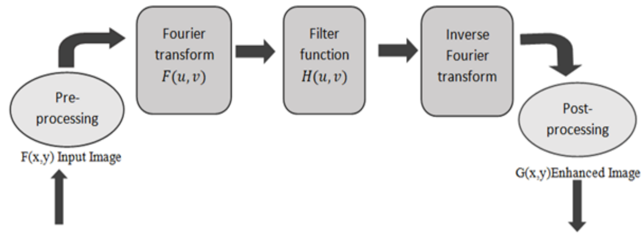

Fig.2 Structured image processing using Fourier Transform

The frequency domain method of image enhancement are depend on convolution theorem. The function shows in Fig.2. $H(u,v)$ is called transfer function where used for boosting the edges of input image $f(x,y)$ to emphasize the high frequency components.

5 Implementation

The experimental evolution is performed on medical image of size 573*893 pixels, as shown in Fig.3. The MATLAB software is operated on a machine with Pentium 4 processer with 512 MB RAMS and clock speed of 1.83 GHZ. Fig.3 presented the original image (3D). Simplifying the original image to 2D Grayscale before starting image processing as displayed in Fig. 4. The membership functions can be used in fuzzy logic approach for image processing, to specify the degree for which pixel is part of an edge. Using the fuzzy logic in command line for edge detecting of the image showed in Fig. 5 to Fig. 8. The intensity of the image threshold and the contrast adjustment (darkest and brightest points in the image) are presented in Fig.9. The process of centering the four angles of the image by using specific algorithm and applying the Fast Fourier transform to facilitate the image enhancement, as introduced in Fig.10. The Results of the implementing the Fast Fourier transform to improve the clarity of the image showed in Fig.11. The concerned image in Fig.11, has processed by applying of the linear filters and non-linear filters to enhance the quality of the image, and the effect of transformation are showed in in Fig.12 & Fig.13.

Fig.3 Original image

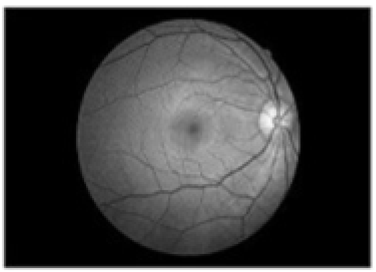

Fig.4 Grayscale image

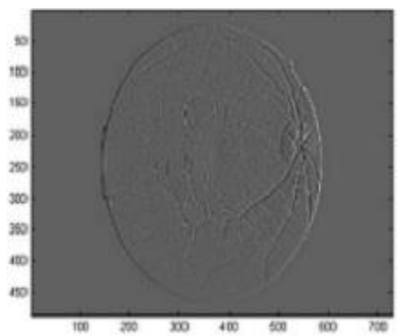

Fig.5 Gx Simple Gradient

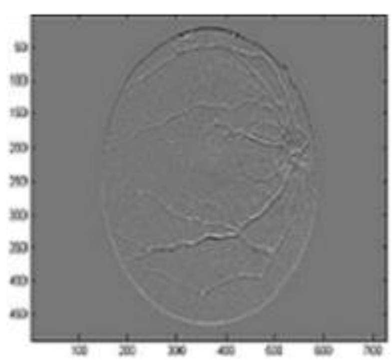

Fig.6 Gy Simple Gradient

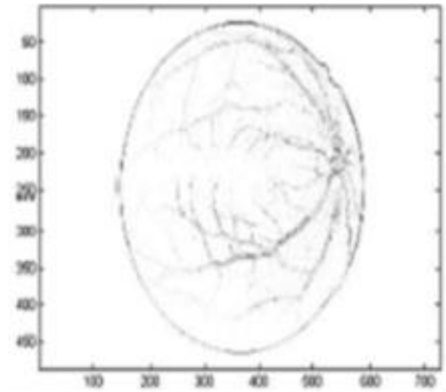

Fig.7 Fuzzy Edge Image

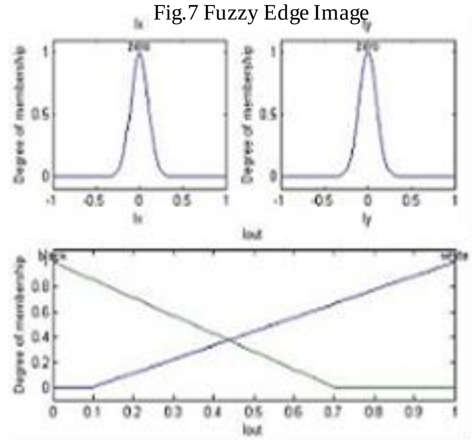

Fig.8 Membership Functions of the Inputs/Outputs of Edge FIS

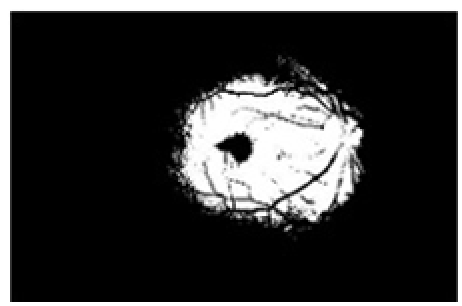

Fig.9 Threshold image

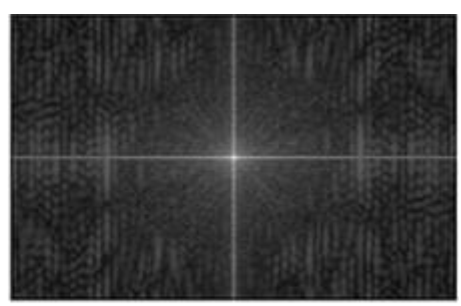

Fig.10 centered Fourier

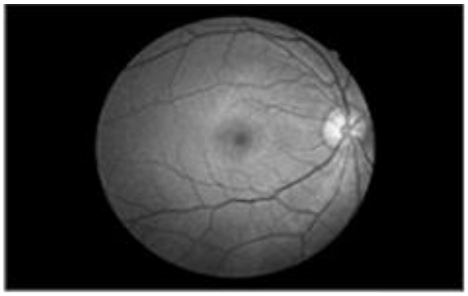

Fig.11 Result of image

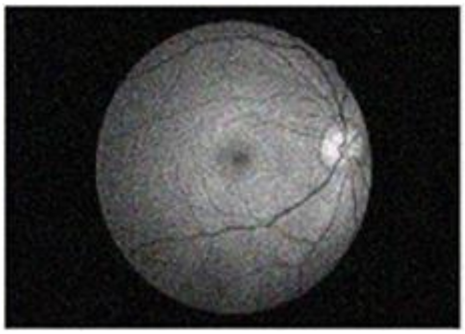

Fig.12 Apply linear filter (Gaussian)

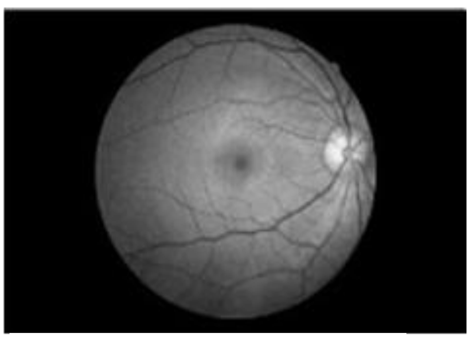

Fig.13 Apply non-linear filter (Median)

6 Discussion and Conclusion

The results viewed the importance of using mathematics in other sciences and its applications especially in image process. Performing various transforms, which are recognized as effective techniques, can solve a wide range of difficulties in image processing. The ability to transform into domain is an essential tool in many applications. This paper suggests using linear and the frequency nonlinear spatial filters in conjunction with frequency domain (Fast Fourier transform) in image processing. A computer program called MATLAB and the Fourier theory that supported by a mathematical function, are employed for processing medical image. The selected filters have applied to the image, after implementing the Fast Fourier transform to improve the quality of the image; and each applied filter showed different effect on processed image. The results showed that the applying of linear filters have less improvement on the processed image. However, non-linear filters indicated a unique distinction in the image-filterable. Finally the resolution presented improvement comparable to the original image as well as small pixels are easily identified.

References

Alejandro Gomez, Echavarria, Juan P Ugarte, Catalina Tobon. (2020) "The fractional fourier transform as biomedical signal and image processing tool". Biocyberentics and biomedical Engineering 40(3), 1081-1093.

Bickey K S, vansh K, Rohan R, Sakil A, Anshul S.(2020) " Evaluation and Comparative study of edge detection techniques". IOSR Journal of Computer Engineering 22(5), 6-15

Frank Dernocourt, (2013). Introduction of fuzzy logic.Massachustts Institute of Technology, 21, 50-56.

García, J., Molina, J. M., Besada, J. A., & Portillo, J. I. (2005). A multitarget tracking video system based on fuzzy and neuro-fuzzy techniques. EURASIP Journal on Advances in Signal Processing, 2005(14), 1-18.

Gómez-Echavarría, A., Ugarte, J. P., & Tobón, C. (2020). "The fractional Fourier transform as a biomedical signal and Image Processing Tool": A Review. Biocybernetics and Biomedical Engineering, 40(3), 1081–1093.

Jiang, Y., & Chai, T. (2015, December). Image Processing and Analysis Based on Partial Differential Equation Method. In 2015 4th National Conference on Electrical, Electronics and Computer Engineering (pp. 985-990). Atlantis Press.

Kaur, R., & Singh, A. (2019). Fuzzy logic: an overview of different application areas. Advances and Applications in Mathematical Sciences, 18(8), 677-689.

Kumar, V., & Nanda, M. (2008). Image Processing in Frequency Domain using Matlab®: A Study for Beginners.

Lotfi, Z.(2013) Real-life Applications of fuzzy logic. Hindawi: Publishing Corporation.

Mahashwari, T., &Asthana, A. (2013). Image enhancement using fuzzy technique. International Journal of Research in Engineering Science and Technology, 2(2), 1-4.

Mandwe, A. A., &Anjum, A. (2016). Detection of brain tumor using k-means clustering. International Journal of Science and Research (IJSR), 5(6), 420-423.

N. Kanwal, A. Girdhar, L. Kaur and J. S. Bhullar, "Detection of Digital Image Forgery using Fast Fourier Transform and Local Features," 2019 International Conference on Automation, Computational and Technology Management (ICACTM), London, UK, 2019, pp. 262-267,

Panchi Li, Hong Xiao.(2018) "An improved filtering method for quantum color image in frequency domain". International journal of theoretical physics 57(1), 258-278,.

Patané, G. (2020). Continuous Fuzzy Transform as Integral Operator. IEEE Transactions on Fuzzy Systems, 29(10), 3093-3102.

Perić, N. (2015). Fuzzy logic and fuzzy set theory based edge detection algorithm. Serbian Journal of Electrical Engineering, 12(1), 109-116.

Pratt, W.K. (2013). Introduction to digital image processing. CRC press.

R Kaur and A Sinch, (2019). Fuzzy logic: An overview of different application areas.In advanced and applications in mathematical sciences.18(8), 677-689.

S. Shi, R. Yang and H. You, "A new two-dimensional Fourier transform algorithm based on image sparsity," 2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), New Orleans, LA, USA, 2017, pp. 1373-1377

Souverville, S., Rosales, J. A., Funes, F. J. G., Dehesa, M., Hernández, I. V., & Lozano,L. V. (2015). Fuzzy Logic Applied to Improvement of Image Resolution using Gaussian Membership Functions. Res. Comput. Sci., 102(1), 77-88.

Timoth, J.(2010). Fuzzy logic with engineering applications. University of new Mexico, USA: Wiley.

Tonyyan Ni, Ruoxu Zhou, chunping Gu, Yang Yang, (2020) " Measurement of Concrete Crack Features with android Smartphone APP based on Digital image Processing Techniques". Vol(150).107093.

Vani, G., &Hema, A. (2020). Energy Consumed Multi Level Feature Extraction on Esophageal X-Ray Images Based on Fuzzy Four Level Fast Fourier Transform. Journal of Green Engineering, 10, 10581-10599.

Vese, L., &Wittman, T. (2010). An introduction to mathematical image processing. Under graduate summer school.

Vinay A, Eddie Yin-Kwee Ng, Arshdeep Singh. (2022) "Machine Learning and its Applications". In Smart Electrical and Mechanical Systems" 1-37.