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Modeling and Estimation of Flow Boiling System Using Kalman Filtering

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Abstract

The Kalman filter (KF) is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. In this paper, the mathematical model of a continuous and discrete flow boiling system has been developed. Kalman filtering was used to estimate the states of a linearizead boiling system. Extended Kalman filtering (KF) was applied to estimate the states of the nonlinear boiling system. It follows from the obtained results that the KF and EKF do the job. KF gives a good result due to optimality and structure. Since it is difficult to install sensors inside the boiler, it will be more convenient to design a state feedback controller using the Kalman filter for the system to track some desired set points (e.g.. Temperature, pressure, etc). The results were presented using MATLAB-Simulink simulations.

Keywords: *Induction machine, diagnostics, current spectrum, harmonics.*

1. Introduction

Kalman filtering is a state estimation technique invented in 1960 by Rudolf E. Kálmán [1,2,3,4.9]. The Kalman filter had already many “spectacular” applications; for example, it was crucial for the Apollo flights to the moon. It gives good results in practice due to optimality and structure, convenient form for online real time processing, easy to formulate and implement given a basic understanding, and measurement equations need not be inverted. Although there are many presentations of Kalman filtering in the literature [10,11] , they are usually focused on particular problem domains such as linear systems with Gaussian noise or robot navigation, which makes it difficult to understand the general principles behind Kalman filtering. This paper describes the procedure of using Kalman filtering to estimate the states of a linear and nonlinear mathematical model for a continuous flow boiling system. The nonlinear mathematical model is developed from the mass and heat balance equations. The system was linearized around the operating point using a Taylor approximation and Simulink method.

1.1 The Phenomenon of Boiling System [1]

The phenomenon of boiling system is such that the temperature responds only to the total pressure P and the vapor flow only to the heat flux q. This leads to the more convenient mode shown in Fig.1-1. The heat balance is used to establish the vapor flux, whereas the system pressure P indicates the temperature. In most cases the differential term $d/dt(VcT)$ is very small in comperation with q and can be neglected. In summary, the only way to change the temperature in case of boiling a single component liquid, is to change the total pressure. Changing the heating rate, this changes only the rate of evolution of vapor. The cause-and-effect relationships for single boiling fluid can be status as:

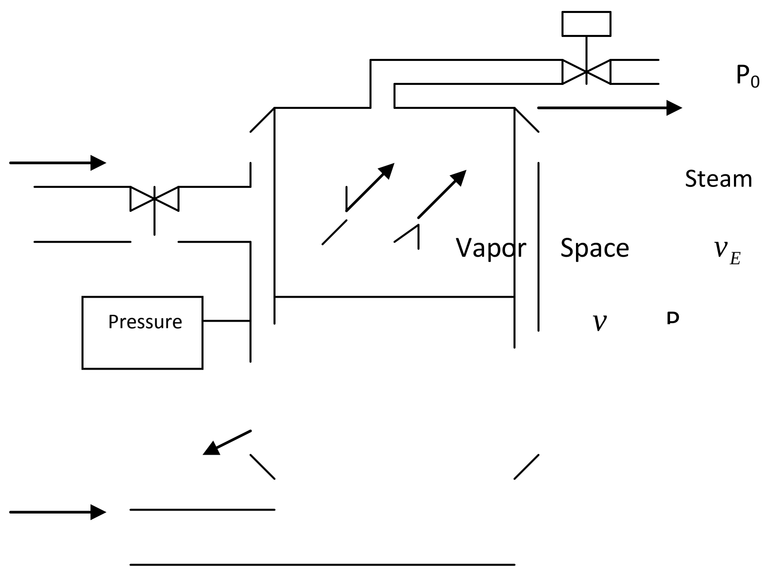

Figure 1-1. Continuous Flow Boiling system.

- Pressure (P) establishes the boiling temperature (T);
- Heat flux (q) establishes the vapor rate (v).

The aim of this paper is to estimate the states of a continuous flow boiling system using Kalman Filtering. The paper is organized into five sections, a reference and an appendix parts. The outline of the paper is as follows:

Section 2, describes the procedure for developing the non-linear mathematical model for a continuous flow boiling system employing the mass and heat balance equations and the linearization of the identified model around the operating point using a Taylor approximation.

Section 3, introduces the theory of the Kalman filtering and extended Kalman filtering.

In section 4, the simulation results of the measurement update and time update equations of a linear and nonlinear system using Matlab, and simulink are made.

Section 5 concludes the results of our paper, and gives future plans.

2. Mathematical Model

2.1 Procedure for Assembling the Non-Linear Model

The procedure for assembling the nonlinear model is as follow:

A. Boundary values:

1. Inlet flow: F_1;
2. Inlet temperature: T_1;
3. Jacket steam pressure: P_S;
4. Exit pressure: P_0;

B. Equations:

1. Valve: $v_E = K\sqrt{P(P-P_0)} \to v_E$ (2-1)
2. Gas law: $PV = m_G RT \to P$ (2-2)
3. Vapor mass balance: $\frac{dm_G}{dt} = v - v_E \to m_G$ (2-3)
4. Boiling point: $T = f(P) \to T$ (2-4)
5. Jacket heat: $q = UA(T_S - T) \to q$ (2-5)
6. Heat balance:

$$\frac{d}{dt}(VcT) = F_1 cT_1 + q - (cT + \lambda)v \to v \qquad (2\text{-}6)$$

7. Mass balance on liquid: $\frac{dV}{dt} = F_1 - v \to V$ (2-7)

8. Gas volume: $V_G = V_0 - \frac{V}{\phi} \to V_G$ (2-8)

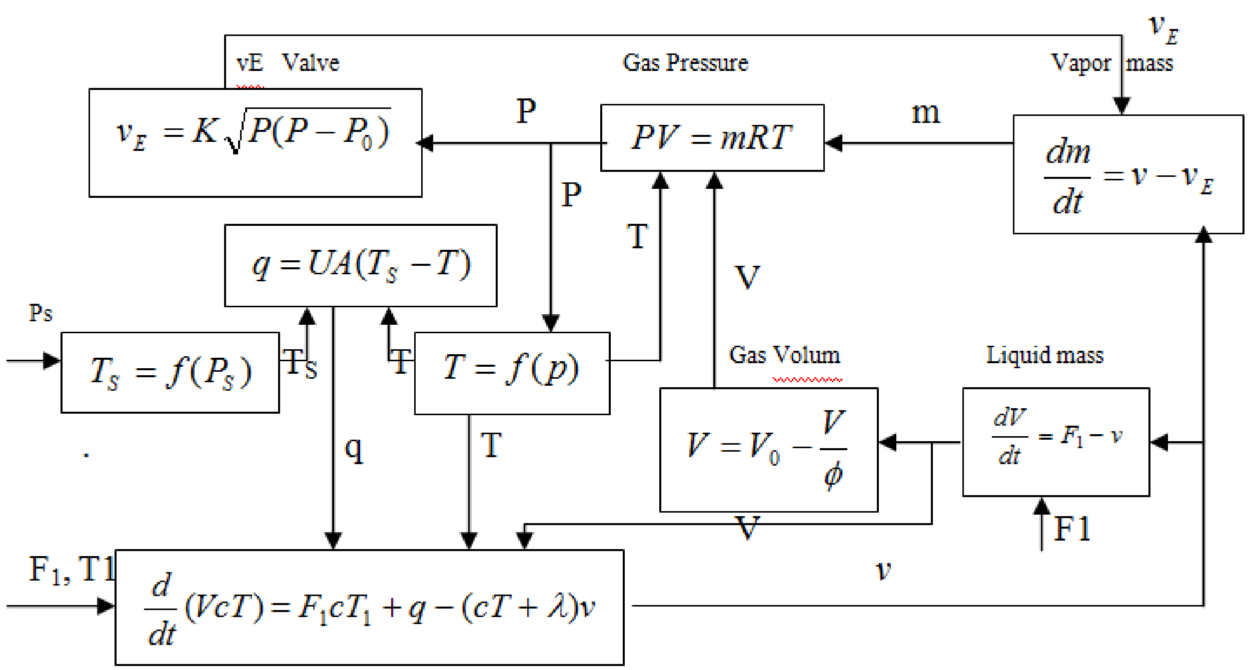

Figure 2-1. Model for continuous flow Boiling system.

The equations described above are assembled in Fig.2-1.

2.2 Building the Non-Linear Model

The mathematical model (in MIMO case) could be built up considering the following variables:

- Input variables:
  - Inlet temperature: $u_1=T_1$;
  - Jacket temperature: $u_2=T_S$;
  - Exit pressure: $u_3= P_0$;

- State Variables:

  Vapor pressure: X_1=P;

  Boiling temperature: X_2=T;

  Vapor mass: X_3= m_G;

- Output variables:

  Temperature: $Y_1 = X_2 = T$;

  Outlet vapor flow rate: $Y_2 = v_E$;

  Vapor mass $Y_3 = X_3 = m_G$;

- Constants:

R=1.98moles/ ft^3,c1=13.96 $KJoule/(Kmol.^{O}C)$,c2=-5210.6 $KJoule/(Kmol.^{O}C)$,V_G=30000 ft^3 , λ=9717 PCU/mole, UA=1700, K=5.7 $ft^3/(Kmol.\text{sec.})$

While the general form of a state space description of a mathematical model is:

$$\begin{aligned}\dot{\bar{X}} &= A\bar{X} + B\bar{u} \\ \bar{Y} &= C\bar{X} + D\bar{u}\end{aligned} \quad , \qquad (2\text{-}9)$$

$$\text{Where: } \bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix} \quad , \bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix} \text{ and } \bar{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} T \\ v_E \\ P \end{bmatrix}. \qquad (2\text{-}10)$$

From the above equations, the state space representation of the non-linear model will be :

$$\dot{X}_1 = \left(\frac{R(X_2+273).X_1.(\ln X_1 - c_1)^2}{V.X_1.(\ln X_1 - c1)^2 + c_2 R X_3}\right)\left(\frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)}\right)$$

$$\dot{X}_2 = \left(\frac{-c_2 R(X_2+273)}{V.X_1.(\ln X_1 - c1)^2 + c_2 R X_3}\right)\left(\frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)}\right)$$

$$\dot{X}_3 = \frac{UA(u_2 - X_2)}{X_2 - u_1 + \lambda} - K\sqrt{X_1(X_1 - u_3)}$$

$$Y_1 = X_2$$

$$Y_2 = K\sqrt{X_1(X_1 - u_3)} \quad .$$

$$Y_3 = X_1 = P \ . \qquad (2\text{-}11)$$

2.3 Linearization of **the Non-Linear Model**

Considering the importance of the operating point [1], the following conditions will be applied to the system:

- The liquid level is maintained at a fixed position by a level controller. This makes V and V_G constants and also the feed flow $F_1 = v_E$;
- The liquid is initially cold and heated up to its boiling point. After boiling starts, the pressure rises to its equilibrium level, raising the temperature to a higher value.

At equilibrium point (operating point) the vapor mass will be constant, therefore:

$$\frac{d}{dt}(m_G) = v - v_E = 0 \quad , \qquad (2\text{-}12)$$

As a result, the following numerical results are obtained:

$u_{1o}=T_1=15$ ºC $X_{1o}=P=1.68301$ atom

$u_{2o}=T_S=150$ ºC $X_{2o}=T=114.71$ ºC (2-13)

$u_{3o}=P_0=1$ atom $X_{3o}=m_G=65.7711$ (unit of mass),

Using the Taylor series expansion and Matlab simulink, the state space representation of the linearized model is obtained:

$$\dot{\bar{X}} = \begin{pmatrix} -0.173873 & -0.00480453 & 6.00964\times10^{-8} \\ -2.98081 & -0.0823552 & 1.03012\times10^{-6} \\ -6.28936 & -0.173797 & 0 \end{pmatrix}\bar{X} + \begin{pmatrix} 0.0000172152 & 0.00478744 & 0.123679 \\ 0.000295089 & 0.0820623 & 2.12 \\ 0.00062272 & 0.173174 & 4.47378 \end{pmatrix}\bar{u}$$

$$\bar{Y} = \begin{pmatrix} 0 & 1 & 0 \\ 6.29 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\bar{X} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -4.47 \\ 0 & 0 & 0 \end{pmatrix}\bar{u} \qquad (2\text{-}14)$$

Where: $\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} P \\ T \\ m_G \end{bmatrix}$, $\bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_S \\ P_0 \end{bmatrix}$, and $\bar{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} X_2 \\ v_E \\ X_3 \end{bmatrix} = \begin{bmatrix} T \\ v_E \\ m_G \end{bmatrix}$. (2-15)

2.4 Discretization of the continuous Model

The discrete state space representation of the boiling system:

$$X_{K+1} = \Phi X_K + \Gamma u_K$$
$$Y_K = HX_K + Du_K$$

$$\Phi = \begin{bmatrix} 0.9184 & -0.0023 & 0 \\ -1.3990 & 0.9613 & 0 \\ -2.9520 & -0.0816 & 1 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} -0.0000 & 0.0023 & 0.0580 \\ 0.0001 & 0.0385 & 0.9949 \\ 0.0003 & 0.0813 & 2.1000 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 6.2890 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4.4740 \\ 0 & 0 & 0 \end{bmatrix} \qquad (2\text{-}16)$$

3. Kalman Filtering Theory

3.1 Kalman Filtering

A Kalman filter is an optimal estimator - i.e. infers parameters of interest from indirect, inaccurate and uncertain observations. It is recursive so that new measurements can be processed as they arrive. If all noise is Gaussian, the Kalman filter minimizes the mean square error of the estimated parameters. Given the mean and standard deviation of noise, the Kalman filter is the best linear estimator [5,6,7]. Our model:

$$X_{K+1} = \Phi X_K + \Gamma u_K + \omega_K$$
$$Z_K = HX_K + Du_K + v_K \qquad (3\text{-}1)$$

Where:

_ X_0 is a Gaussian random variable with mean X_0 and covariance P_0.

_ ω_K is a zero mean white noise random sequence with covariance Q.

_ v_K is a zero mean white noise random sequence with covariance R.

_ X_0, ω_K and v_K are mutually independent.

If x, and z are jointly Gaussian random vectors with

$$E[x] = \bar{x} \text{ , and } E[z] = \bar{z}$$

$$E\left[\begin{bmatrix} x-\bar{x} \\ z-\bar{z} \end{bmatrix} \begin{bmatrix} x-\bar{x} \\ z-\bar{z} \end{bmatrix}^T\right] = \begin{bmatrix} P_x & P_{xz} \\ P_{zx} & P_z \end{bmatrix} \qquad (3\text{-}2)$$

Then the conditional distribution of x given z is:

$$P_{X/Z}(x/z) = N(\hat{x}, P_{X/Z})$$

Where:

$$\hat{x} = \bar{x} + K(z - \bar{z})$$

$$K = P_{XZ} P_Z^{-1}$$

$$P_{X/Z} = P_X - P_{XZ} P_Z^{-1} P_{XZ}^{\ T} \qquad (3\text{-}3)$$

Given the initial state and covariance, we have sufficient information to find the optimal state estimate using the Kalman filter equations. The Kalman filter equations can be obtained as follows:

Given: $X_o \sim \left(X_0^{(-)}, P_0^{(-)}\right)$

- The Measurement Update:

$$\bar{K}_k = P_k^{(-)} H^T (H P_k^{(-)} H^T + R)^{-1}$$

$$X_K^{(+)} = X_k^{(-)} + \bar{K}_k \left(z_k - \left(H X_k^{(-)} - D u_k \right) \right)$$

$$P_k^{(+)} = (I - \bar{K}_k H) P_k^{(-)} \quad (3\text{-}4)$$

- The Time Update:

$$X_{k+1}^{(-)} = \Phi X_k^{(+)} + \Gamma u_k$$

$$P_{k+1}^{(-)} = \Phi P_k^{(+)} \Phi^T + Q \quad (3\text{-}5)$$

3.2 Extended Kalman Filtering

Many practical systems have non-linear state update or measurement equations. The Kalman filter can be applied to a linearized version of these equations with loss of optimality [8].

Our model:

$$\begin{aligned} X_{K+1} &= f(x_k, u_k) + \omega_K \\ Z_K &= g(x_k, u_k) + v_K \end{aligned} \quad (3\text{-}6)$$

Given: $X_o \sim \left(X_0^{(-)}, P_0^{(+)} \right)$

Let $$H_K = \left. \frac{\partial g}{\partial x_K} \right|_{x_k = x_k^{(-)}}$$

- The Measurement Update:

$$\bar{K}_k = P_k^{(-)} H^T (H P_k^{(-)} H^T + R)^{-1}$$

$$X_k^{(+)} = X_k^{(-)} + \bar{K}_k \left(z_k - g(\hat{x}_k^{(-)}, u_k) \right)$$

$$P_k^{(+)} = (I - \bar{K}_k H) P_k^{(-)}$$

Let $$\Phi_K = \left.\frac{\partial f}{\partial x_K}\right|_{x_k = \hat{x}_k(-)} \qquad (3\text{-}7)$$

- The Time Update:

$$X_{k+1}^{(-)} = f(\hat{x}_k^{(+)}, u_k)$$

$$P_{k+1}^{(-)} = \Phi P_k^{(+)} \Phi^T + Q \qquad (3\text{-}8)$$

4. Simulation and Results

We considered the following values for our process and measurement noise:

$$Q = \begin{bmatrix} 0.5^2 & 0 & 0 \\ 0 & 0.2^2 & 0 \\ 0 & 0 & 0.5^2 \end{bmatrix} \quad , \quad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Kalman filtering equations were simulated using the Matlab Simulink (A.1). with the following initial states and covariance:

$$\hat{X}_0^{(-)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad , \quad P_0^{(-)} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

4.1 Simulation of the Linear Model Using KF

Using the Kalman filter equations (3-4), and (3-5), the linearized model of the boiling system was simulated and the following results were obtained as shown in figures (4-1),(4-2), and (4-3).

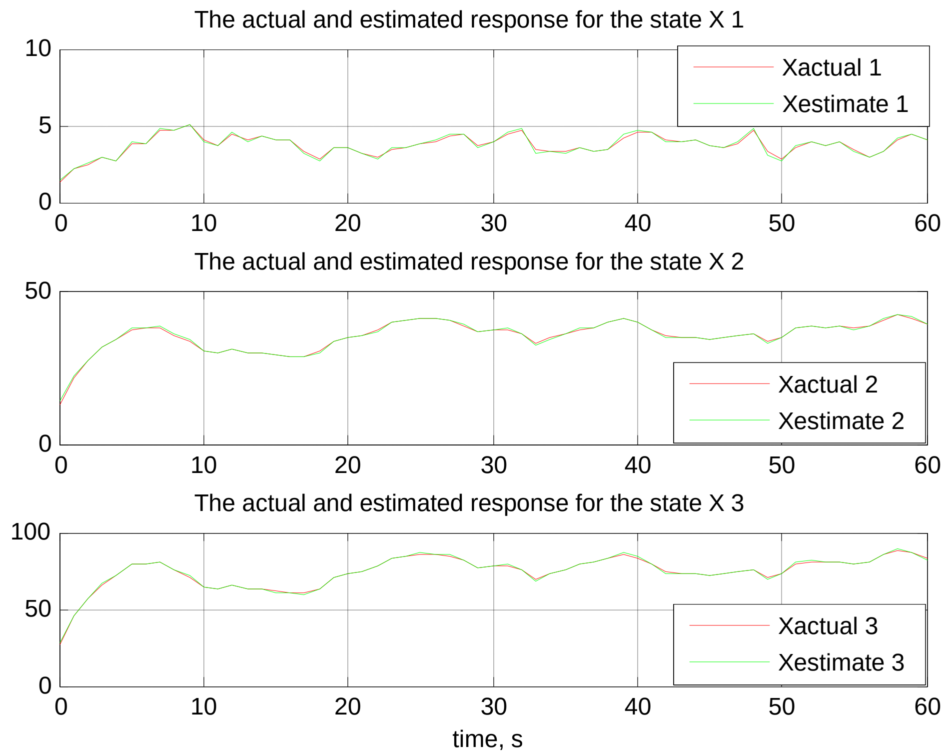

Figure 4.1 The actual and estimated response for the linear states

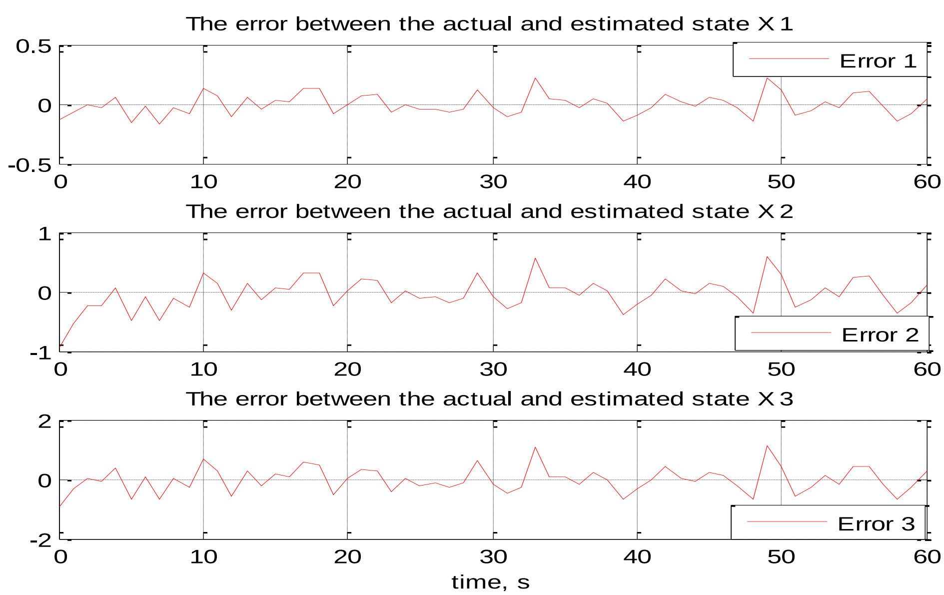

Figure 4.2 The error between the actual and estimated of the linear states

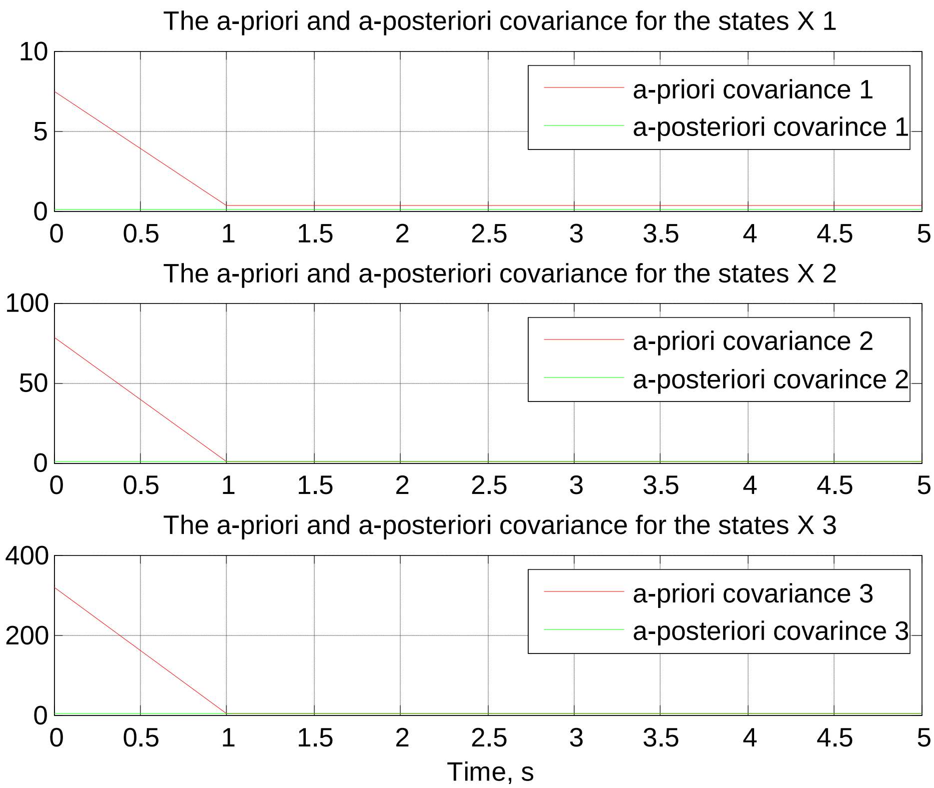

Figure 4.3 The a-prior and a-posteriori covariance for the linear states

Table (4-1): The first five iterations for KF simulation

	Actual Values			**Estimates Values**			**Error a-Post. Cov.**		
	Vapor Press.	**Boiling Temp.**	**Vapor Mass**	**Vapor Pressure**	**Boiling Temp.**	**Vapor Mass**	**Vapor Pressure**	**Boiling Temp.**	**Vapor Mass**
k	**X1**	**X2**	**X3**	**X1**	**X2**	**X3**	P_{11}	P_{22}	P_{33}
0	1.326	12.968	27.450	1.452	13.902	28.366	0.023	0.911	0.954
1	2.126	21.399	45.282	2.195	21.925	45.585	0.023	0.494	0.660
2	2.491	27.004	57.110	2.503	27.236	57.064	0.023	0.371	0.608
3	2.914	31.298	66.192	2.948	31.530	66.261	0.023	0.320	0.594
4	2.731	33.955	71.741	2.668	33.888	71.378	0.023	0.295	0.588

5	3.760	37.381	79.098	3.913	37.868	79.790	0.023	0.023	0.585

As it can seen from the above figures and table, the Kalman filtering gives a good results due to optimality and structure.

4.2 Simulation of **the Non-Linear Model** Using EKF

Using the extended Kalman filter equations (3-7), and (3-8), the nonlinear states of the boiling system were estimated and the following results were obtained as shown in figures (4-4), (4-5), and (4-6).

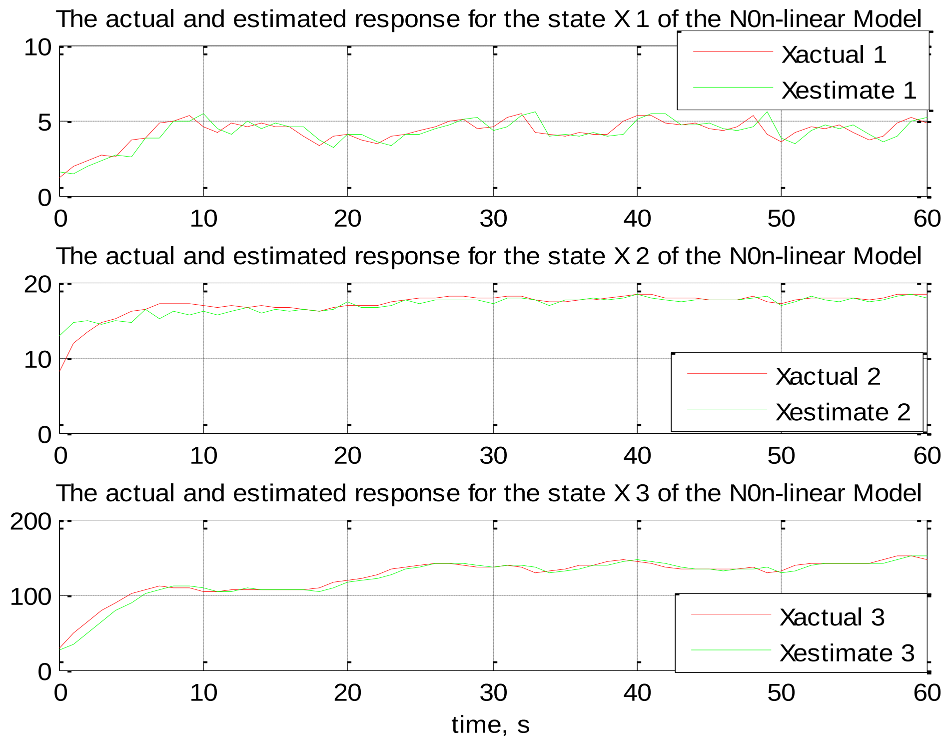

Figure 4.4 The actual and estimated response for the nonlinear states

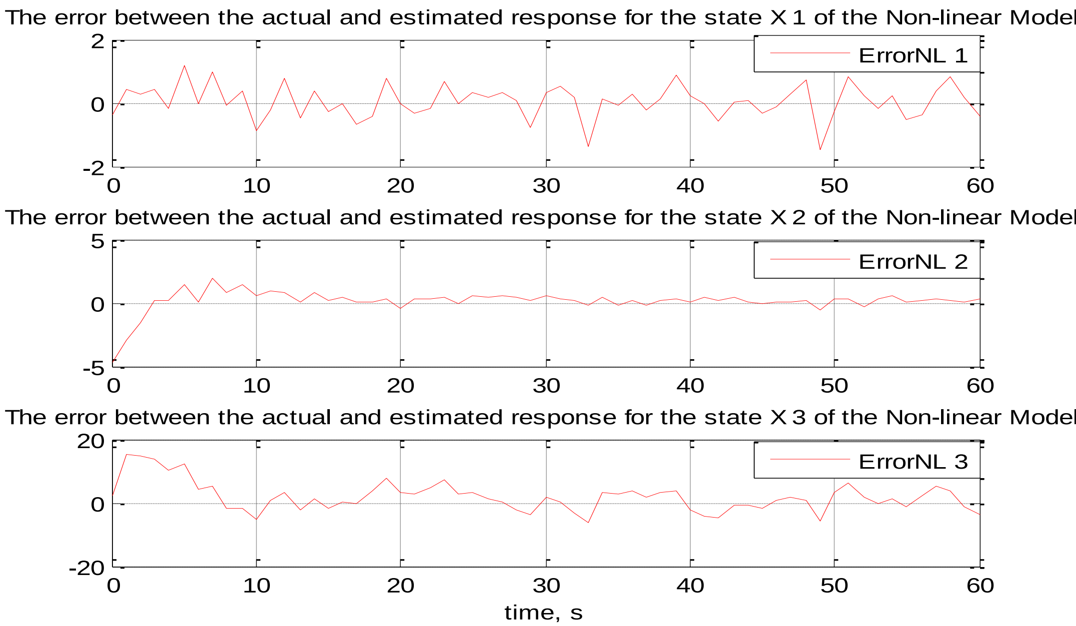

Figure 4.5 The error between the actual and estimated of the nonlinear states

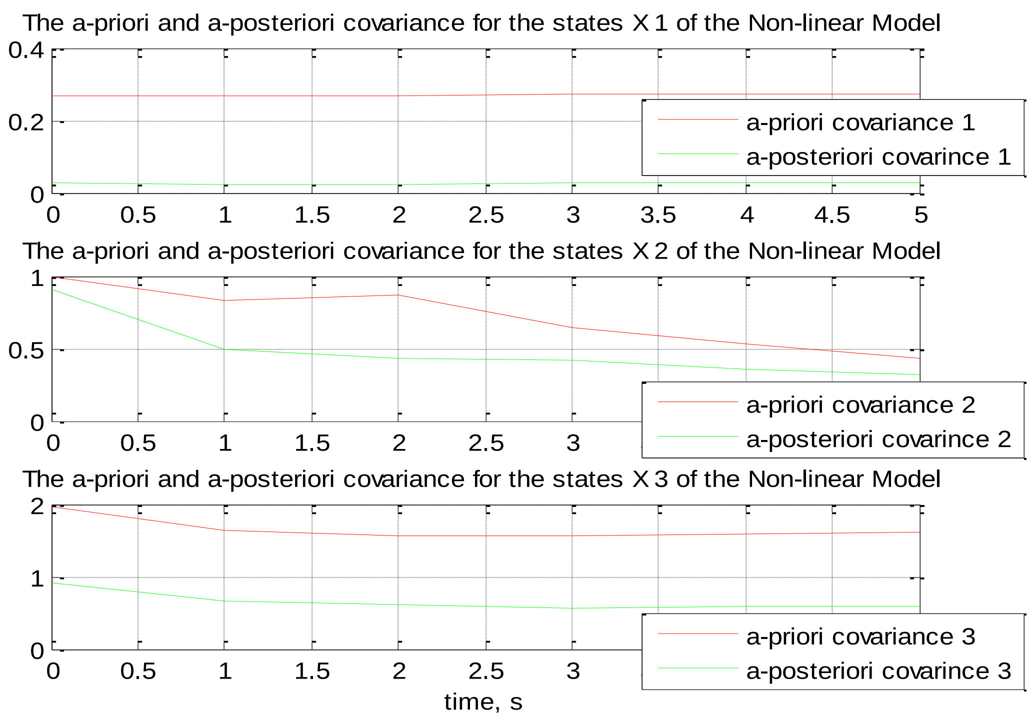

Figure 4.6 The a-prior and a-posteriori covariance for the nonlinear states

Table (4-2): The first five iterations for EKF simulation

	Actual Values			Estimates Values			Error a-Post. Cov.		
	Vapor Press.	Boiling Temp.	Vapor Mass	Vapor Pressure	Boiling Temp.	Vapor Mass	Vapor Pressure	Boiling Temp.	Vapor Mass
k	**X1**	**X2**	**X3**	**X1**	**X2**	**X3**	P_{11}	P_{22}	P_{33}
0	1.152	8.223	28.396	1.508	12.849	26.014	0.025	0.909	0.909
1	1.889	11.810	48.993	1.476	14.696	33.716	0.024	0.491	0.653
2	2.255	13.397	64.644	1.998	14.935	49.762	0.024	0.425	0.600
3	2.715	14.510	78.166	2.276	14.285	64.396	0.025	0.420	0.574
4	2.595	15.087	88.470	2.749	14.889	78.079	0.026	0.358	0.581
5	3.687	16.103	100.522	2.530	14.624	88.125	0.026	0.317	0.590

As it can seen from the above figures and table, the extended Kalman filtering gives a good results due to optimality and structure.

5. Conclusions

The mathematical model of a continuous flow boiling system has been developed. Using Kalman filtering, the states of a linearizead boiling system were estimated. The results were presented using MATLAB-Simulink simulations. Using Extended Kalman filtering, the states of the nonlinear boiling system were estimated the results were presented using MATLAB-Simulink simulations. It follows from the obtained result that the KF, and EKF do the job. KF gives a good result due to optimality and structure. Since it is difficult to install sensors inside the boiler, it will be more convenient to design a state feedback controller using the Kalman filter for the system to track some desired set points (e.g.. Temperature, pressure, etc).

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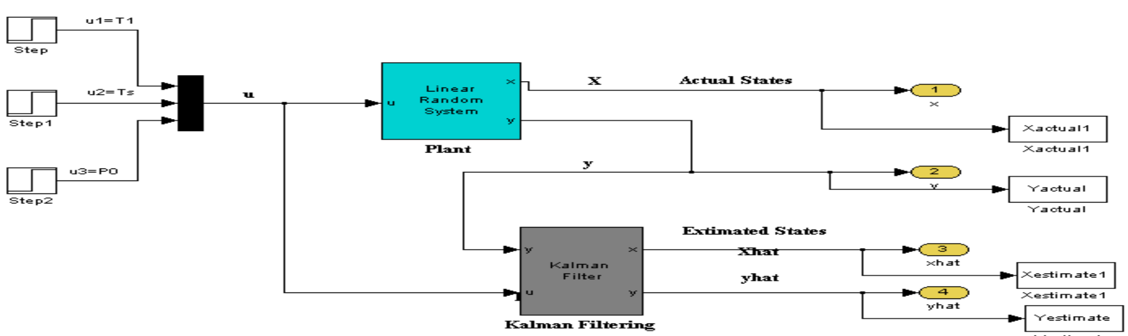
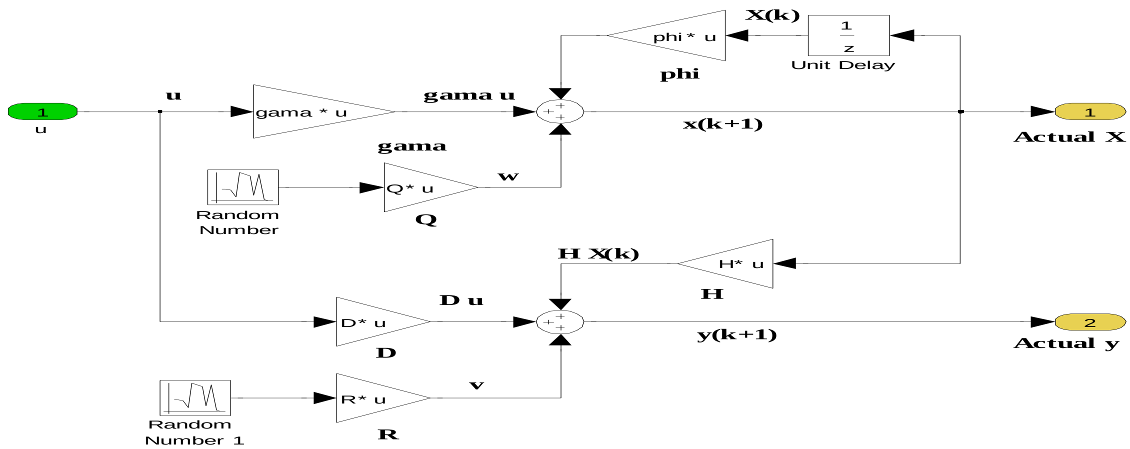
Appendix

A.1.1 Simulation of the linearized continuous flow boiling system using the Kalman filtering.



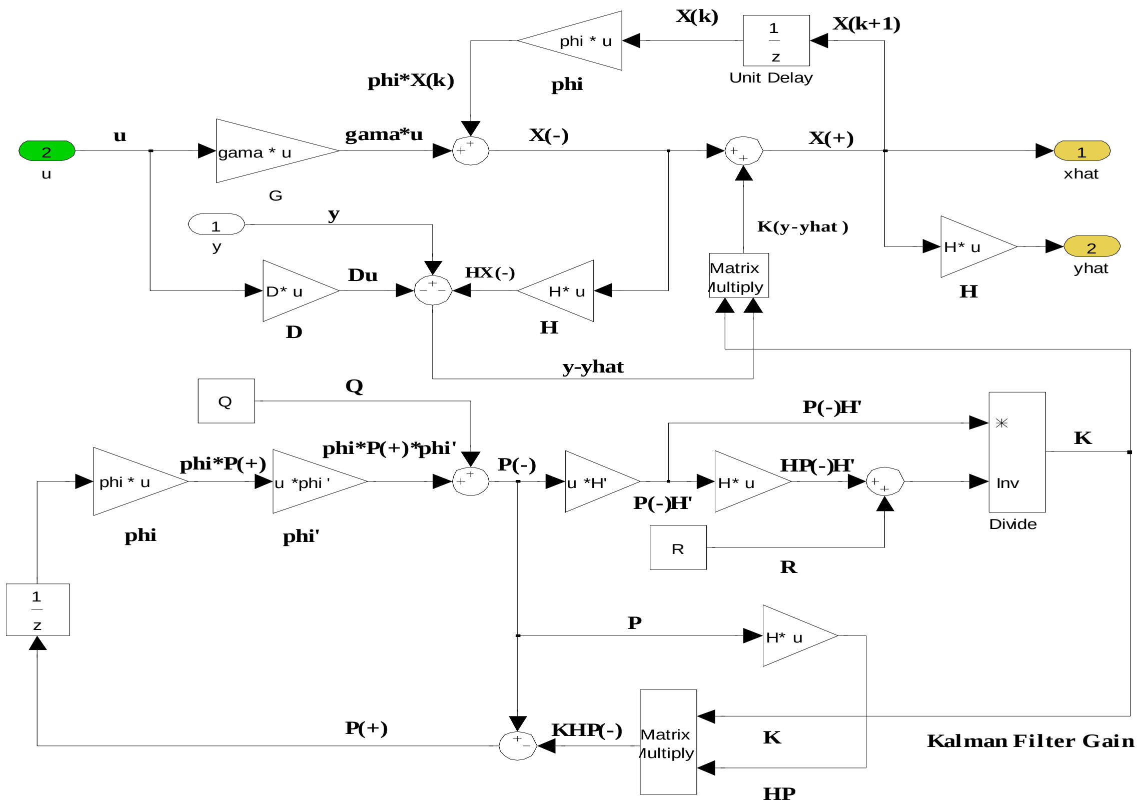
Simulation of the linearized continuous flow boiling system using the Kalman filtering.

A.1.2 The simulink model of the Plant Equations



The simulink model of the Plant Equations

A.1.3 The simulink model of the Kalman Filter Equations



The simulink model of the Kalman Filter Equations