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Adaptive Control of Discrete Repetitive Processes in Iteration Domain

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Abstract

In this paper, the problem of model reference adaptive control for unit memory discrete repetitive processes is analyzed and solved by employing a lifting technique that allow us to view the discrete repetitive processes as a first-order multivariable plant. An adaptive controller gain adjustment algorithm in iteration domain is given that ensures the monotonic convergence of the tracking error between the process and the desired reference model outputs under persistent excitation conditions.

Key Words: *Repetitive processes, Model reference adaptive control, Iteration domain.*

1. Introduction

There is a considerable amount of literature on continuous-time single-input single-output (SISO) adaptive control algorithms [1], [2]. However, the global stability and convergence of these algorithms has been studied under general assumptions [3], [4]. Model Reference Adaptive Controller using state variables is proposed for a class of multi-input multi-output (MIMO) systems under assumption that one matrix is known [5]. The structural flexibility of discrete repetitive processes (DRPs) representation that produced from applying the “supervectors” and lifting technique in [6] motivated us to extend the model reference adaptive control schemes for discrete (MIMO) systems in the literature to (DRPs). Many problems in control theory are characterized by systems that have a repetitive nature, whereby the operation of the system to be controlled is repeated in some fashion. One such class of problems is iterative learning control (ILC) [7], [8].

Another class of problems with a repetitive nature is defined by a plant given by [9]

$$x(k + 1, t + 1) = Ax(k + 1, t) + Bu(k + 1, t) + B_0y(k, t)$$

$$y(k + 1, t) = C x(k + 1, t) + Du(k + 1, t)+ D_0y(k, t) \quad (1)$$

where x(k, t) is the state, u(k, t) is the input, y(k, t) is the output. It is clear that (1) has dynamics in two dimensions: a time, or along-the-pass, dimension, t, also simply called the time axis, and an iteration dimension k, also referred to as the repetition or iteration axis.

Here we see a two-dimensional aspect, with evolution in time t and iteration k, as in ILC. But, in ILC systems the iteration-domain variation arises due to the controller, whereas in (1) the iteration-domain memory in the system is inherent in the plant. Such systems are called unit-memory processes [9] (meaning that only information from the most recent iteration is used in the plant). The continuous version of (1) is called a differential repetitive process while (1) is called a discrete repetitive process. In this paper we will restrict our attention to unit-memory discrete repetitive processes, or DRPs.The goal of the controller is to improve the operation from trial-to-trial (or pass-to-pass or iteration to iteration).

Repetitive processes have been studied for a number of years, perhaps arising first in the application of long-wall coal mining. This problem, together with a number of other repetitive processes that arise in applications, has been described in [9], which presents a comprehensive analysis of the system- theoretic properties of repetitive processes, including stability, controllability, and observability. Notably, it is pointed out in [9] that ILC is in fact a repetitive process, a point also captured in the ILC framework presented in [7].

In [9] a number of results are provided for feedback controller design for repetitive processes. Here we add to these results by considering the adaptive controller case. First, we use a lifting technique and the definition of an iteration-domain frequency operator to change the two-dimensional single-input, single-output plant into a one-dimensional multivariable system. Next it is shown how to describe a closed-loop DRP process as a multivariable control system. Then, taking the advantage of the structural flexibility matrices that produced to develop a systematic MRAC design strategy that ensures perfect tracking along the pass and asymptotically-convergent tracking along the iteration axis. Two examples are then considered to illustrate the results via analysis and simulations : designing a model reference adaptive controller to track a desired DRP output and an estimator to estimate the markov parameters of DRP in iteration domain that required by the former design.

2. Drp Representations

2.1 DRPs in the 2-D Time and Iteration Domains

Consider again the single-input, single-output (SISO) DRP (1) for $0 \leq t \leq N$, where the positive integer t is the independent variable associated with the "along-the-pass" axis, N is the pass length, and the positive integer $k \geq 0$ is the pass number, or independent variable associated with the iteration axis. We assume there exists an initial boundary condition y-1 (t) and that xk (0) = x0 for every k.

A system such as (1) can also be usefully described in the complex frequency domain (using usual z-transform notation) as

$$Y(k+1) = H^{u}(z)U(k+1,z) + H^{Y}(z)Y(k,z) \quad (2)$$

where $Y(k,z)$ and $U(k,z)$ are the z-transforms of $y(k,t)$ and $u(k,t)$, respectively, and

$$H^{u}(z) = C(zI - A)^{-1}B + D$$

$$H^{Y}(z) = C(zI - A)^{-1}B_0 + D_0$$

2.2 DRPs as 1-D Iteration-Domain Systems

As described above, the DRP is a two-dimensional (2-D) SISO system. However, we can convert it into a one-dimensional (1-D) multiple-input, multiple-output (MIMO) system by first defining so-called "supervectors" as

$$U(k) = (u(k,0), u(k,1), \cdot\cdot\cdot, u(k,N-1))^T$$

$$Y(k) = (y(k,0), y(k,1), \cdot\cdot\cdot, y(k,N-1))^T$$

We can then give the lifted representation of the system as

$$Y(k+1) = H^{u}U(k+1) + H^{Y}Y(k) \quad (3)$$

Where H^{u} and H^{Y} are lower-triangular Toeplitz matrices of rank N whose elements are the Markov parameters of the system, given by:

$$H^u = \begin{bmatrix} h_0^u & 0 & \dots & 0 \\ h_1^u & h_0^u & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1}^u & h_{N-2}^u & \dots & h_0^u \end{bmatrix} \quad \text{and} \quad H^y = \begin{bmatrix} h_0^y & 0 & \dots & 0 \\ h_1^y & h_0^y & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1}^y & h_{N-2}^y & \dots & h_0^y \end{bmatrix}$$

Where $h_0^u = D,\ h_0^y = D_0$ and for $i \geq 1,\ h_i^u = CA^{i-1}B,\ h_i^y = CA^{i-1}B_0$

2.3 DRPs in the Iteration-Frequency Domain

As defined in [10], for each t ∈ [0, N], let the (one-sided) w-transform W (·) be given as

$$W(\{u(k.t)\}) = \sum_{k=0}^{\infty} u(k.t)w^{-1} \qquad (4)$$

As we have noted in earlier work, the w-transform is similar to the standard z-transform, but it is operating from trial-to-trial, with time t fixed, as opposed to the standard z-transform operator, which operates from time step-to-time step, with k fixed. The first formal definition of this operator was given in [10], but it was first introduced in [6] and its use in ILC has been given in a number of places, including [11], [12], [13].

Two useful properties of the w-transform are:

1) Shift Property [10]: Assuming $u_j = 0$ for all j < 0, we can use (4) to write
W ({u(k - n, t)}) = w^{-n} W ({u(k, t)})
2) Final Value Theorem: For a signal (supervector) X_k we have

$$\lim_{k\to\infty} X(k) = \lim_{w\to 1} X(w+1)X(w)$$

when the limit exists (e.g., if X (k) is "stable"). Applying the w-transform to our lifted vectors U (k) and Y (k), to get U (w) and Y (w), respectively, and applying the shift property, we may write the DRP (3) as

$$W(\{Y(k+1)\}) = W(\{H^u U(k+1) + H^y Y(k)\})$$

$$wY(w) = wH^u U(w) + H^y Y(w)$$

or

$$Y(w) = w(wI - H^y)^{-1} H^u U(w) \qquad (5)$$

$$= \; wH^{u}(wI - H^{y})^{-1}U(w)$$

The latter expression due to the fact that lower-triangular Toeplitz matrices commute.

3. Non Adaptive Controller of Drps

In conventional DRP theory, as described in [9], a number of control algorithms have been posed, including:

1) Memory-less state or output feedback:

$$u_{k+1}(t) = F\,x_{k+1}(t) + Gy^{d}(t)$$

where F, G are matrices and y^{d} (t) is the reference trajectory.

2) "PI"-like dynamic pass profile controller:

$$u_{k+1}(t) = k_1 u_{k+1}(t) + K_2(z)\sum_{j=1}^{k} e_j(t) \qquad (6)$$

where Ki (z) are time-domain filters.

Considerable analysis has been carried out on such algorithms, including H∞-based design for the case of the dynamic pass profile controller. However, we note that the structure of all these algorithms is similar in that only the dynamic pass controller makes use of input data from past trials (that is, no terms u_k (t), u_{k-1} (t), etc. appear in the update for u_{k+1} (t)) and only the so-called "PI" form uses the error of the trajectory. Also, though we note that one can achieve stability and tracking with the memory-less state feedback equation, this result will not be robust, as we demonstrate below.

Thus, motivated by ILC thinking, we find it natural to consider algorithms that explicitly use the error of the trajectory as well as information about the both the input and the error from multiple past trials. Specifically, as in a typical ILC algorithms, consider using filtered errors from m previous passes as well as the current pass and filtered inputs from n ≥ m previous passes, resulting in an algorithm of the form:

$$u_{k+1}(t) \;=\; -\bar{D}_{n-1}(z)u_k(t) - \cdots - \bar{D}_0(z)u_{k-n+1}(t)$$

$$+ \; N_m(z)e_{k+1}(t) + \cdots + N_0(z)e_{k-m+1}(t) \quad (7)$$

Where $\bar{D}_i(z)$ and $N_i(z)$ denote discrete-time filters with the usual abuse of notation and the error is defined as ek (t) = yd (t) − yk (t). Note that we admit the possibility of an iteration-varying reference and that with the exception of the current pass factor Nm(z), all these filters may be non-causal (in time).

To proceed, we apply the lifting and w-transform machinery introduced in the previous section to (7) in the usual way (see [10], for example), getting first

$$U_{k+1} = -\bar{D}_{n-1}U_k - \cdots - \bar{D}_0 U_{k-n+1} \tag{8}$$

$$+ \; N_m E_{k+1} + N_{m-1}E_k + \cdots + N_0 E_{k-m+1},$$

which becomes, after taking the w-transform of both sides of this equation, applying the shift property, and combining terms,

$$\bar{D}_C(w)U(w) = N_C(w)E(w),$$

We can then write the relationship between U (w) and E(w) as a matrix fraction:

$$U(w) = \bar{C}(w)E(w) \tag{9}$$

Where $\bar{C}(w) = \bar{D}_c^{-1}(w)N_c(w)$

4. Model Reference Adaptive Control of Drp

Figure 1 shows the proposed structure of the model reference adaptive control of the discrete repetitive processes in the iteration domain (w-domain). Using the supervectors the lifted representation of the discrete repetitive process is rewritten for convenience:

$$Y_p(k+1) = H^u U(k+1) + H^y Y_p(k) \tag{10}$$

and the transfer matrix of the plant in w-domain is

$$G_p(w) = w(wI - H^y)^{-1}H^u \tag{11}$$

where $H^u \in R^{N\,xN}$ and $H^y \in R^{N\,xN}$ are unknown lower-triangular Toeplitz matrices of rank N whose elements are the Markov parameters of the system as described in section (2.2) and it is assumed that only the sign of the first Markov parameter H^u is known.

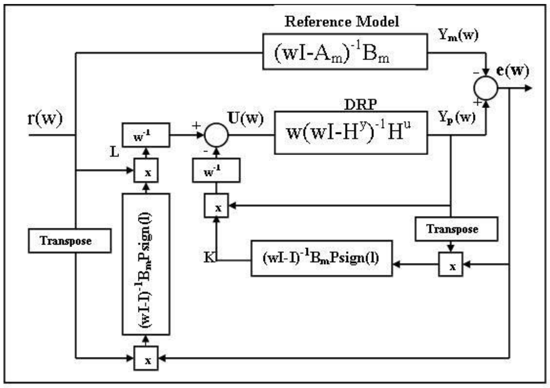

Figure 1. The MRAC structure of DRPs in w-domain.

The reference model which describes the desired behavior of the DRP, is characterized by the linear time invariant system

$$Y_m(k+1) = B_m\, r(k) + A_m\, Y_m(k) \tag{12}$$

and the transfer matrix of the reference model in w-domain is

$$G_m(w) = (wI - A_m)^{-1}B_m \tag{13}$$

where $A_m \in R^{N\,xN}$ is a known and asymptotically stable matrix, Bm ϵ RN xN is a known matrix, and r is a N-dimensional iteration invariant input vector with bounded elements or r(k) iteration-varying input reference [14].

4.1 The Control Law

The control objective is to choose the input vector $U(k) \in R^N$ such that all signals in the closed-loop plant are bounded and the plant output $Y_p(k) \in R^N$ follows the output $Y_m(k) \in R^N$ of a reference model where N is the pass length and the tracking error

$$e(k) = Y_p(k) - Y_m(k) \tag{14}$$

monotonically tends to zero as the iteration $k \to \infty$. The control U to the plant, is generated introducing control law

$$U(k+1) = -KY_p(k) + Lr(k) \tag{15}$$

where $K \in R^{N\,xN}$ and $L \in R^{N\,xN}$ are the estimates of the controller matrix gains K* and L* respectively, to be generated by an adaptive law. It is worth to note that the control law (15) is a special case of the general ILC, it directly follows by substituting equation (10) in equation (15) that

$$U(k+1) = \bar{D}U(k) + N\,E(k) \qquad (16)$$

where $D^{-} = -K\,H^{u}$ and $N = K\,H^{y} = L$, which is ILC like algorithm. Using equation (15) and equation (11) the closed loop transfer matrix (plant and controller) becomes

$$G_{cl}(w) = [wI - (H^{y} - H^{u}K)]^{-1}H^{u}L \qquad (17)$$

In order to achieve the control objective, we need to guarantee the existence of K* and L* such that the equations

$$H^{y} - H^{u}K^{*} = A_m, \qquad H^{u}L^{*} = B_m \qquad (18)$$

are satisfied. Then the transfer matrix of the closed-loop plant (17) is the same as that of the reference model (13) and Yp (k) → Ym (k) exponentially fast for any bounded reference input signal r.

It is worth to mention that in general, K*, L* might not be existed to satisfy the matching condition in equation (18) for the given matrices, implying that the control law (15) may not have enough structural flexibility to meet the control objective. In contrast to our case where the super-vectors technique formulated the DRP matrices in lower- triangular Toeplitz which give the enough structural flexibility to ensure the existence of the solution of equation (18) regardless of the invertibility of the matrix H u in case of (H_0^u).

4.2 The Adaptive Law

The controller parameters are to be directly or indirectly generated by an appropriate adaptive law. In this paper the direct MRAC method has been used, where the plant is to be parameterized in terms of the controller parameters. By extending the approaches that used for the single input-single output (SISO) continuous systems in [1] and for the multivariables continuous systems (MIMO) in [5], to the systems in the iteration domain and by choosing the A_m and B_m of the reference model such that the transfer matrix Gm (w) in (13) is strictly positive real matrix, the following adaptive law follows:

$$\begin{bmatrix} K(k+1) \\ L(k+1) \end{bmatrix} = \begin{bmatrix} K(k) \\ L(k) \end{bmatrix} + \begin{bmatrix} B_m^T Pe(k)sign(l) & 0 \\ 0 & B_m^T Pe(k)sign(l) \end{bmatrix} \omega \qquad (19)$$

Where: $\omega = \left[Y^T(k) \quad -r^T(k) \right]^T$,

$$sign(l) = \begin{cases} 1 \ \ if \ \ L^{*} \ is \ positive \ definite \ matrix \\ -1 \ \ if \ \ L^{*} \ is \ negative \ definite \ matrix \end{cases}$$

$e(k) = Y_p(k) - Y_m(k)$

and $P = P^T > 0$, satisfies $A_m^T P A_m - P = -QQ^T$.

for some $Q = Q^T > 0$. Further, if the signal vector ω(k) is "persistently exciting", then

$K(k) \to K^*$ and $L(k) \to L^*$ as $k \to \infty$.

Or $\lim_{k\to\infty} \|\Phi(k)\| = 0$.

where $\Phi(k) = [\ K(k) - K^* \quad L(k) - L^*]^T$.

4.3 Stability: Convergence of The Tracking Error

Subtracting equation (12) from (10), and using equations (15), (18), the error equation is obtained as

$e(k + 1) = Am\ e(k) + Bm\ L^{*-1} (-\Phi T\ (k)\omega(k))$ (20)

then the error transfer matrix

Ge (w) = (wI − Am)−1Bm = Gm (w) (21)

The proof of stability follows directly by using a lyapunov function candidate

V (e, Φ) = eT P e + tr(ΦT ΓΦ)

from the discrete Kalaman-Szogo-Popov lemma it is known that, if G_m (w) is SPR discrete transfer matrix, then there exist a symmetric positive definite matrix P and matrices M, Q such that

$$A_m^T P A_m - P = -QQ^T,$$

$$B_m^T P A_m + M^T Q^T = C_m^T,$$

$$D_m + D_m^T - B_m^T P B_m = M^T M.$$

compute the change of V (k) along (20) and using (19), we get

$$\Delta V(k) = V(k+1) - V(k) = -e^T QQ^T e \leq 0.$$

Hence, the equilibrium state is uniformly stable and

$$\lim_{k\to\infty}\|e(k)\| = 0.$$

Again it is worth to mention that the structural flexibility (eg. H^u, H^y are lower-triangular Toeplitz matrices) that evolved by applying the super-vectors technique to DRP enabled us to overcome the restriction that the unknown L* in the matching equation $H^u L^* = Bm$ is either positive or negative definite. Since H^u is a lower-triangular Toeplitz matrix, it's inverse is also a lower-triangular Toeplitz.

5. Examples and Simulation Results

To illustrate the results of the MRAC scheme that presented in the previous section, consider the following discrete repetitive plant

$$Y(k+1,z) = H^u(z)U(k+1,z) + H^Y(z)Y(k,z)$$

Where:

$$H^u(z) = \frac{1.2 + 1.24z^{-1} + 0.085z^{-2}}{1 + 0.2z^{-1} + 0.0125z^{-2}}$$

$$H^y(z) = \frac{0.8 + 0.66z^{-1} + 0.09z^{-2}}{1 + 0.2z^{-1} + 0.0125z^{-2}}$$

and with pass length t ∈ [1, 30]. Using the "supervectors" and lifted representation , the system becomes as in (10) with

$$H^u = \begin{bmatrix} 1.2 & 0 & \dots & 0 \\ 1 & 1.2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1.2 \end{bmatrix} \quad \text{and} \quad H^y = \begin{bmatrix} 0.8 & 0 & \dots & 0 \\ 0.5 & 0.8 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0.8 \end{bmatrix}$$

where $H^u \in R^{30x30}$ and $H^y \in R^{30x30}$ are lower-triangular Toeplitz matrices of rank 30 whose elements are the Markov parameters of the system.

The matrices A_m and B_m of reference model which describes the desired plant output were chosen such that the following requirements are met

1) The existence of the solution of the matching eq. (18).

There are many possibilities to choose Am and Bm to satisfy this condition, the simplest one may be to set in the form of $Am = \alpha I^{30x30}$, where $|\alpha| < 1$, so that all the discrete poles of the reference model will be placed inside a unit circle and $Bm = \beta I^{30x30}$, in our example we took $\alpha = -0.2$ and $\beta = 2$ respectively.

2) The control objective.

As a result of the appropriate choices of the matrices of the reference model, the control objective was achieved as illustrated below. In what follows three cases of the reference input to the model (step, time and iteration varying and signal that changes from step to time and iteration varying in iteration domain) will be demonstrated respectively.

Case 1: Constant Reference Input: Consider the following step reference signal in iteration domain as shown in Fig. 2.

$r(t) = 3 + \sin(0.1t) + 1.5 \sin(0.5t)$

The simulation result of applying the above reference signal to the proposed discrete MRAC is plotted in Fig. 3. The plant is assumed initially at rest. As seen in the figure, since the rms error between the reference model and the plant output converges to zero very fast along the iteration axis, the tracking is perfect.

Case 2: Sinusoidal Reference Input: Consider the following signal that varies sinusoidally in time and iteration and takes the following form as shown in Fig. 4:

$r(t, k) = 3 + \sin(0.1t) + 1.5 \sin(0.5t) + 0.3 \sin(0.04k)$

Fig. 5 shows the convergence of the rms error between the model and the plant output along the iteration axis, which indicates that the desired behavior of the plant is achieved before the iteration k=100.

Case 3: Discontinuous Reference Input: To further illustrate the potency of the suggested MRAC for DRPs, we consider the case where the reference signal changes from a constant signal to time and iteration varying signal at k = 50 iteration as given by the following equation and shown in Fig. 6:

$$r(t,k) = \begin{cases} 1 + sin(0.1t) + 2sin(0.3t) & 1 \le k \le 50 \\ 3 + 1.5sin(0.5t) + sin(0.02k) & 51 \le k \le 100 \end{cases}$$

Fig. 7 shows that the proposed scheme can handle this change in the reference input and insure a perfect convergence of the rms error between the model and the plant output along the iteration axis, which implies that the control objective is achieved before the iteration k = 100.

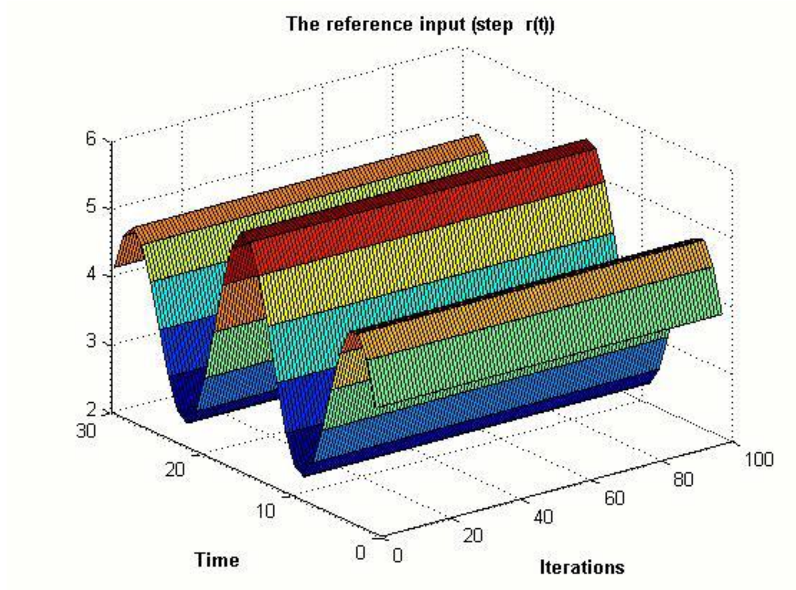

Figure 2. The step reference input in iteration domain of case 1.

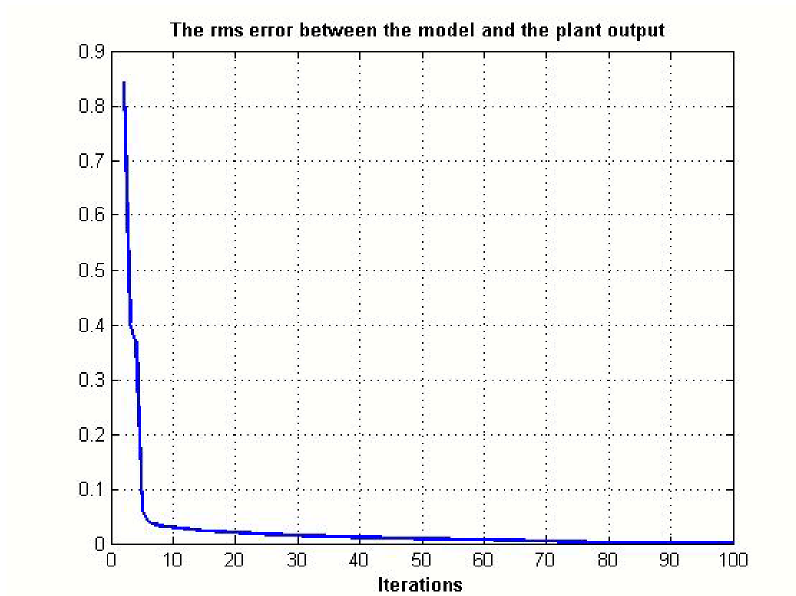

Figure 3. Convergence of the norm of the error of the direct MRAC scheme of DRP of case 1.

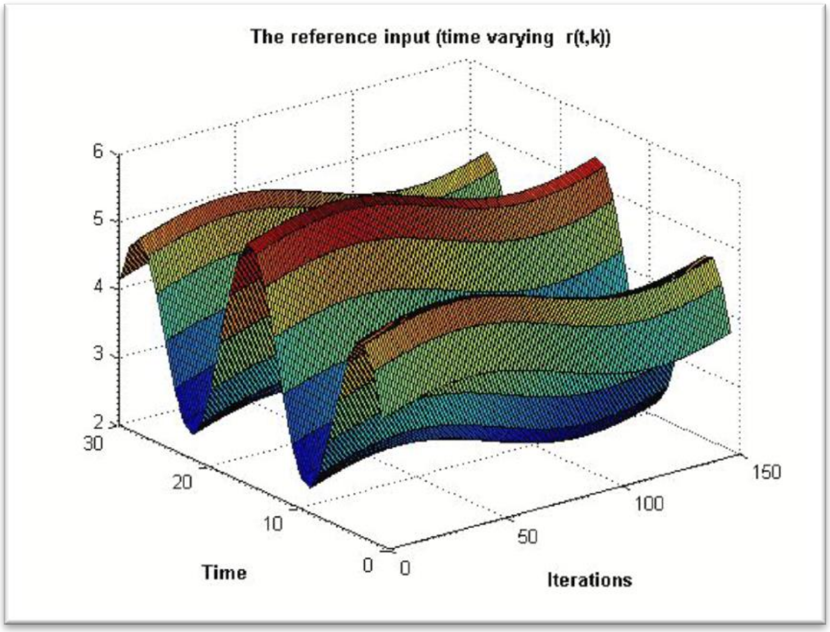

Figure 4. The time varying reference input in iteration domain of case 2.

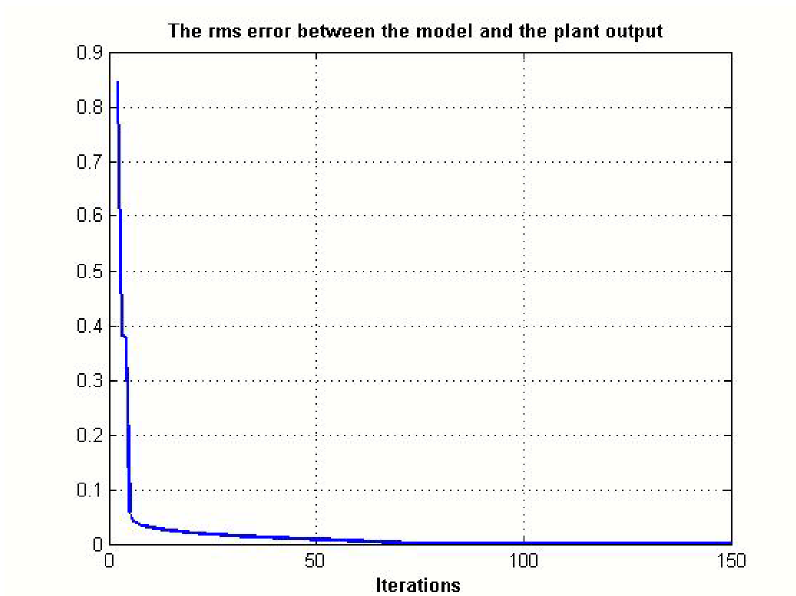

Figure 5. Convergence of the norm of the error of the direct MRAC scheme of DRP of case 2 .

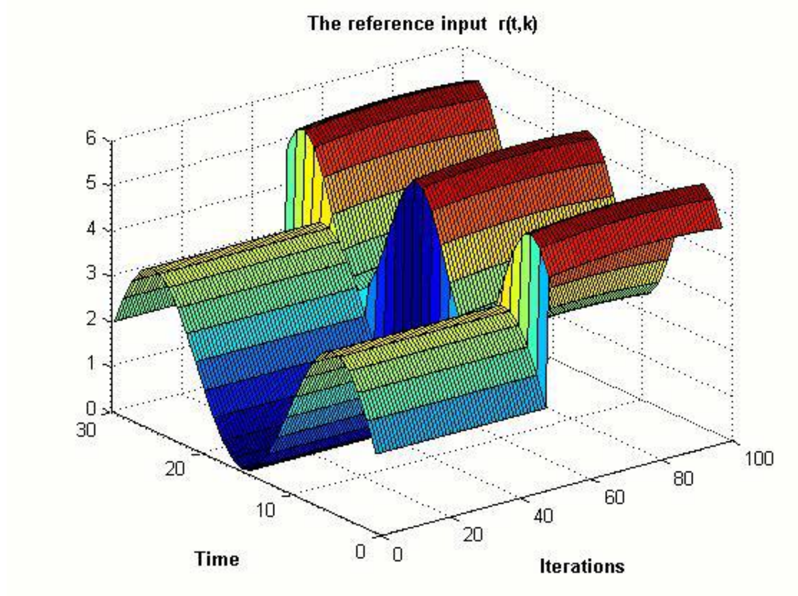

Figure 6. The reference input changes from step to time varying at k = 50 of case 3.

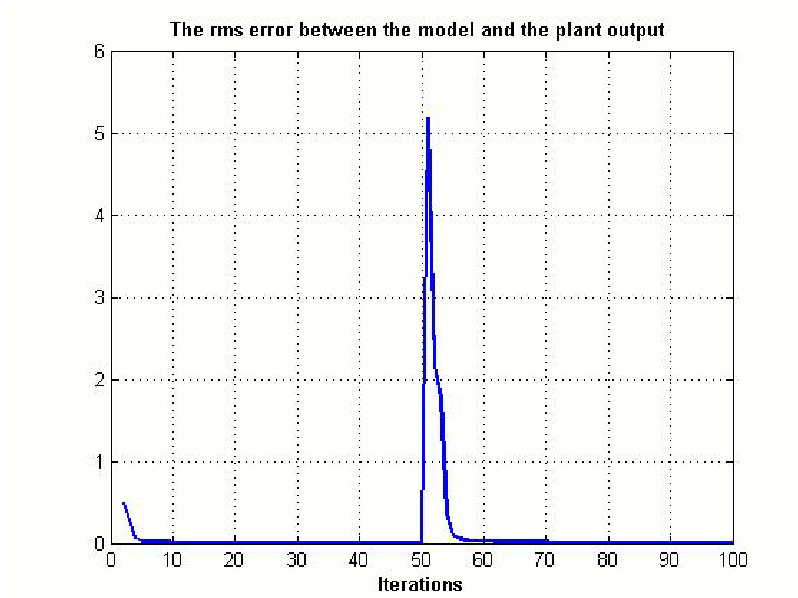

Figure 7. Convergence of the norm of the error of the direct MRAC scheme of DRP of case 3.

6. Conclusion

In this paper we have systematically demonstrated how to use an adaptive control technique for unit memory discrete repetitive processes. Using a lifting technique and an iteration-domain complex frequency operator, the two-dimensional single-input, single-output plant was changed into a one- dimensional multivariable system. Then discrete time multivariable model reference adaptive control scheme was developed, which guarantees uniformly stability and asymptotic output tracking, and it is applicable to DRP models in iteration domain. Simulation results were obtained which as desired, indi- cate that using adaptive control in DRPs is certainly a promising approach for performance guarantees in the presence of system uncertainties.

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