Sirte University Scientific Journal(Applied Sciences) Vol.7(2),01–13,December2017

Modeling Ligaments as Composite Materials

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Abstract

The anterior cruciate ligament (ACL) is one of the four major ligaments of the human knee. It was studied to compute its mechanical properties in 2D and 3D as a composite. ACL ligament structure was studied based on equal stress in the fiber and matrix (Rule of Mixture) and Halpin Tsai model treatment and its effects when modeled as a laminate. The Eshelby method was employed for the calculation of expressions for stresses and strains in composites, and hence their elastic constants. The study looked at how differently it behaved when loading at an angle (Φ) to the fiber axis to understand the way composite behaves when loaded uniaxially and when loaded in other directions. Assuming that there is a perfect bonding between the fiber and matrix at the interface, the ACL has similar behavior to a transversely isotropic composite. Rule of Mixture and Halpin Tsai model was carried out. A value of unity was selected for the adjustable parameter (ξ=1), [1]. The Eshelby method was utilized to compute the Esehlby tensor and the stiffness matrix. The study procedure starts with knowing the Young's modulus and the Poisson ratio of the ground matrix and the fiber [2,3], then the behavior of the Young's modulus of the composite in axial and transverse directions were computed. Besides elastic modulus, Poisson ratio and the shear modulus for a varying fiber volume fraction were also computed. For all additional calculations, a fiber volume fraction of 50% was assumed and computation was performed. The stiffness tensor Cpq and the compliance matrix Spq were appropriately computed for 2D and 3D cases for both Rule of Mixture and Halpin-Tsai models. From the components of the Eshelby tensor matrix, calculations were done to obtain the Eshelby tensor for the composite. It was observed that Rule of Mixture and Halpin-Tsai models followed the same trend with a slight offset in the data. The Eshelby matrices and the stiffness matrix based on Eshelby calculation are presented. This study gives us the opportunity to know how the ACL ligaments elasticity behaves when modeled as a composite comprised of the ground matrix and unidirectional continuous collagen fibers. Results and plots obtained for various parameters against the loading angle can give an estimate of the trend consistent with the change in the loading angle. Variations in the three methods for five transversely isotropic elastic parameters are presented in this study.

Keywords: *Composites, rule of mixture, Halpin Tsai equations, Eshelby method, ACL ligament, elastic modulus.*

1. Introduction

The concept of load sharing between the matrix and the reinforcing constituent (fiber) is central to an understanding of the mechanical behavior of a composite. An external load applied to a composite is partly borne by the matrix and partly by the reinforcement. The load carried by the matrix across a section of the composite is given by the product of the average stress in the matrix and its sectional area. The load carried by the reinforcement is determined similarly. Equating the externally imposed load to the sum of these two contributions, and dividing through by the total sectional area, gives a basic and important equation of composite theory, sometimes termed the "Rule of Mixture"[4]. However, the value is an underestimate, since in practice there are parts of the matrix effectively "in parallel" with the fibers (as in the equal strain model), rather than "in series" as is assumed. Empirical expressions are available, which give much better approximations, such as that of Halpin-Tsai. This project will deal primarily with unidirectional-reinforced fiber composites and with properties measured along and transverse to the fiber direction.

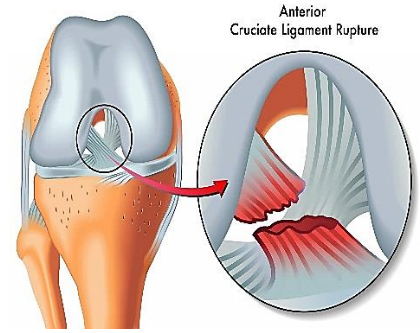
Many composites that used today are at the leading edge of materials technology, with performance and costs appropriate to ultra-demanding applications such as aviation, bioengineering, automobiles, etc. Heterogeneous materials, however, combining the best aspects of dissimilar constituents have been used by nature for millions of years [5]. Ligaments in the human body are basically tough bundles called collagen fibers, which are also a major component of cartilage, tendons, and bones. They support most tissues and give cells structure from the outside. Collagen has great tensile strength [6,7]. These collagen fibers are naturally arranged in a particular orientation throughout the ground substance. The objective of this project is to model ACL ligament as composite material that has transversely isotropic behavior using the Rule of Mixtures, the Halpin-Tsai models, and the Eshelby method. Since unidirectional fibers are rarely seen inside the in vivo, the material parameters were computed for changing loading angles. Material parameters graphs were plotted which showed the trends seen with a change in loading angle.

2. Materials and Methodology

The composite selected for this study is the ACL. It is comprised of the ground matrix (proteoglycans and water) and embedded collagen fibers as shown below. The material properties of Young’s modulus and Poisson ratios were taken from the literature [2,3] which is shown in Table 1.

Table 1. Material properties for the ground matrix and collagen fibers

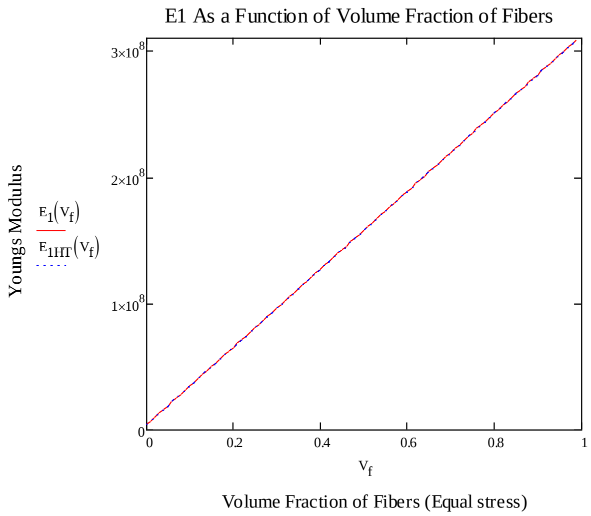
	Collagen fibers	**Ground matrix**
Young's modulus [MPa]	$E_f = 312.5$	$E_m = 4.2$
Poisson ratio	$\upsilon_f = 0.3$	$\upsilon_m = 0.45$



Using MathCAD software (Mathsoft, Cambridge, MA), the Young's modulus of the composite in the axial direction was computed using the Rule of Mixtures. For the estimation of the Young's modulus in the transverse direction, Rule of mixtures and Halpin-Tsai models were employed. The computation of Halpin-Tsai model used the adjustable factor taken as 1 (i.e. $\xi = 1$). Results were plotted with Young's modulus E_1 and E_2 on Y-axis and the varying fiber volume fraction on the X-axis. The shear modulus G_{12} and G_{23} were computed after knowing the shear modulus of the matrix G_m and fibers G_f. Poisson ratio υ_{12}, υ_{21} and υ_{23} calculations were made and values were plotted against the fiber volume fraction. The graphs were categorized as Equal stress and Halpin-Tsai graphs. For the 2D case, the stiffness matrix C_{pq} and the compliance matrix S_{pq} were calculated. Each of the matrix was a 6x6 matrix. All further 2D computations were carried out using customized commercial coding Matlab® software (Mathworks Inc., Nattick, MA)**.** Calculations were made at 50% fiber volume fraction at different loading angles where the rotated stiffness matrix and compliance matrix were $\hat{C}_{pq}$ and $\hat{S}_{pq}$ respectively. Based on these rotated matrices, the elastic constants E_1, E_2, G_{12}, υ_{12}, υ_{21} were calculated. The interaction ratio η_{xyx} and η_{xyy} were computed. Two different sets, one for equal stress and one for Halpin-Tsai, were computed and plotted against the loading angle Φ on the X-axis. A similar procedure was followed for 3D computation where the equations for the compliance matrix and stiffness matrix changed. The trends of the elastic constants were plotted against the loading angle. Using commercial coding software, the Eshelby tensor component equations were calculated and the total stiffness tensor was computed.

3. Results and Discussions

Rule of Mixtures or "equal stress," is an approach to approximate estimation of composite material properties, based on an assumption that consider the composite property is the volume weighed average of the phase (matrix and dispersed phase) properties. E_1 increases almost linearly to a maximum magnitude of 312.5 MPa. E_2 computed using Rule of Mixtures method and Halpin-Tsai behave in almost the same manner. It is shown that there is a sudden increase in the transverse modulus after a 90% fiber volume fraction Fig. 3.1.



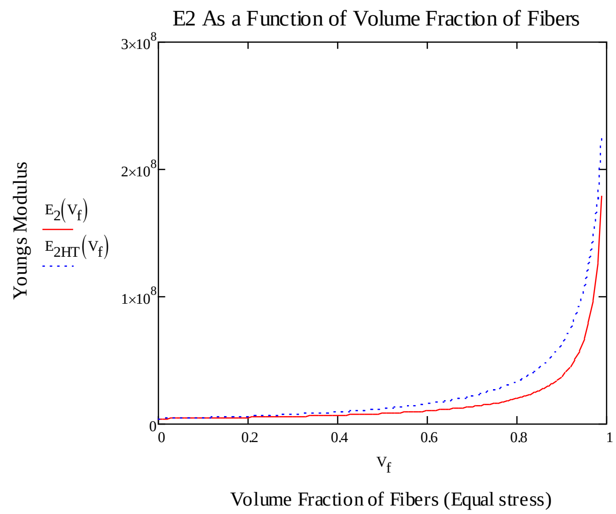


Figure. 3.1 Young's modulus in the axial and transverse directions rule of mixture E_1, E_2 and Halpin-Tsai E_{1HT}, E_{2HT}.

The Poisson ratio was calculated using the rule of mixtures for υ_{12}, υ_{21}, and υ_{23} and results show a linear decreased in their values from 0.45 to 0.3. This may clearly indicate that in the presence or the absence of fibers in the composite they behave similarly with or without reinforcement respectively. The plots in Fig. 3.2 convey an idea of the pronounced tendency under transverse loading for the composite to contract in the other transverse direction in preference to the axial direction. Such effects are of particular significance for the behavior of laminates.

The dependence of the young's modulus and shear modulus of the lamina on the value of the loading angle is shown in Fig. 3.3 using both the equal stress and Halpin-Tsai. The tensile stiffness remains close to the theoretical maximum if the stress axis is within a few degrees of the fiber axis, however if Φ is more than about 5° then it decreases rapidly. The shear stiffness is less sensitive than young's modulus to Φ, but a pronounced peak is always exhibited at around 45°.

There is a number of aspects that are clear in Fig. 3.4. such as considerable changes that take place as the loading angle changes. Interaction ratio is zero at Φ =0 and Φ =90. The effects are significant at intermediate angles. The peak value is -2.834 at Φ =9.2. Using the equal stress assumption for E_2 gives a slight overestimation of the interaction terms.

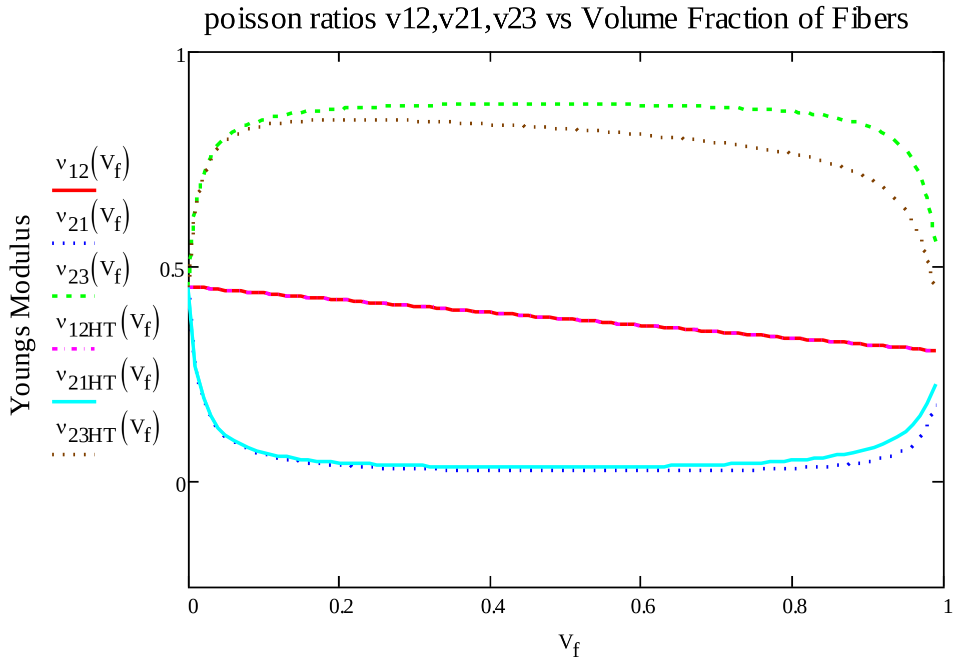


Figure. 3.2: The Poisson ratio was calculated for the rule of mixtures and HalpinTsia for υ_{12}, υ_{21}, $\upsilon_{23.}$ $\upsilon_{12.}$

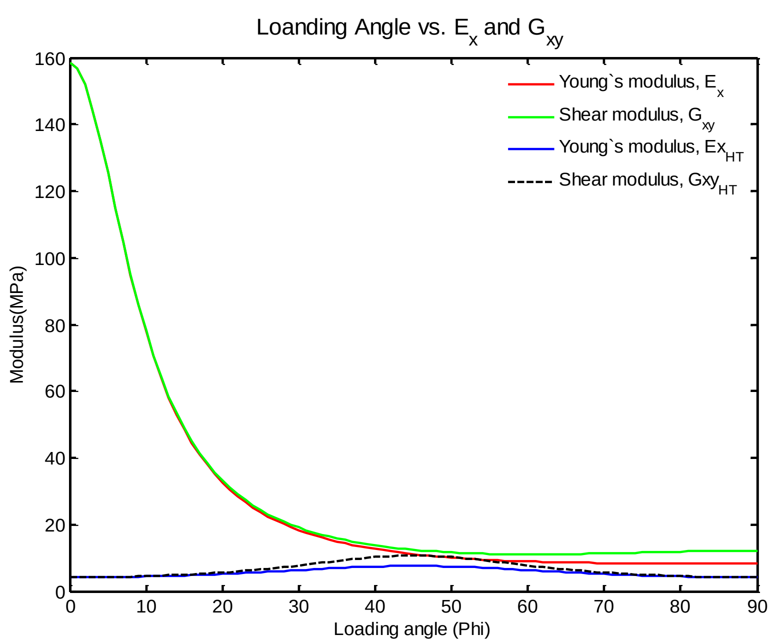


Figure.3.3. Young's modulus E_x and the shear modulus G_{xy} calculated using equal stress and Halpin Tsai method.

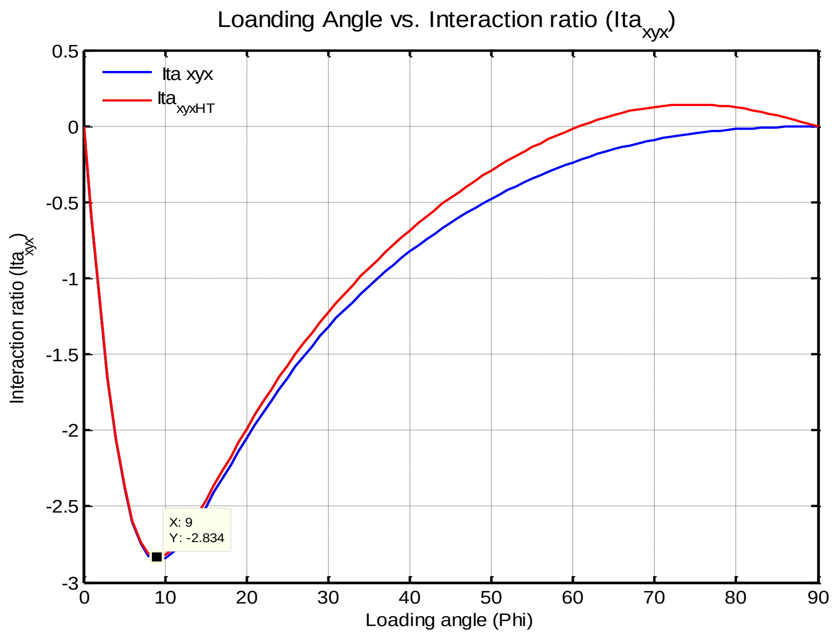


Figure. 3.4. Variation with loading angle Φ of the interaction ratio η_{xyx} for laminae of ACL at f=0.5.

From these estimates, the variation of the shear stress in the matrix at the fiber-matrix interface can be determined. When the composite is loaded at an angle, they show a significant matrix shear strains. These shears are responsible for the effects that is shown in Fig. 3.5, which shows the Poisson ratio υ_{xy}. For the Halpin-Tsai method, it starts with a value of 0.37, it attains a maximum value of 0.57 around 20°. The Poisson ratio drops after 20° and reaches an insignificant Poisson ratio of 0.02 at 90°. A similar tendency is noted for the Rule of Mixtures method with a small measurable difference. It can be observed that large strains in the matrix bring both shear distortions and large lateral contractions. Effects like these are important in understanding the shear behavior of laminates.

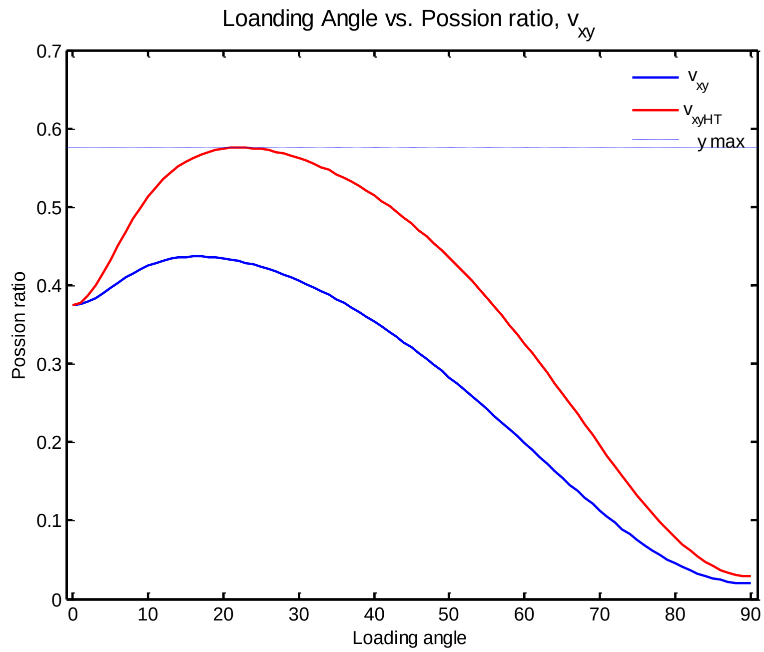


Figure. 3.5: Variation with loading angle Φ of the Poisson ratio υ_{xy} for laminae of ACL at f= 0.5.

Fig. 3.6 shows that the shear strain is directly proportional with the interaction compliance. It can be seen that large shear strain is expected around 20°-40° loading angle.

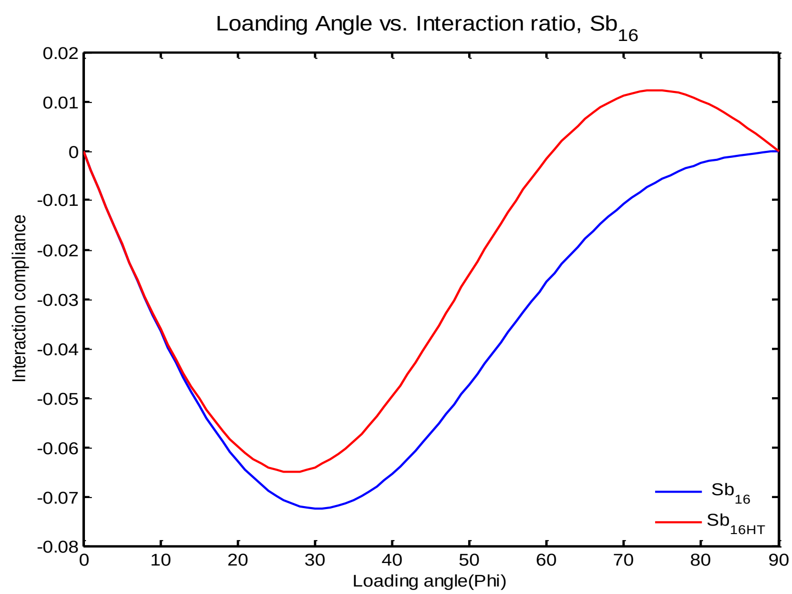


Figure. 3.6. Predicted variation with loading angle Φ of the interaction compliance $\hat{S}_{16}$, for the case of the composite material in Equal stress and Halpin-Tsai methods.

3.2. 3D elastic constants

A number of calculations were in use for the computation of elastic constants for 3D composite. The equations for the stiffness matrix and the compliance matrix varied accordingly. The following graphs in Fig. 3.7 - 3.10, 3D computation has a higher shear component, and since they are transversely isotropic, the behavior is unvaried. For transversely isotropic properties, the modulus E_{11} indicateda maximum value of 158 MPa and with the lowest modulus being 8.3 MPa for a 90° rotation. The shear modulus slightly increased to 4.21 MPa for 0° orientation to 7.59 MPa for an angle of 45°. The interaction ratio η_{xyx} was in the negative quadrant and reached 0 at an angle of 61° before going into the positive quadrant for Halpin-Tsai. Similar results were obtained in the 3D case.

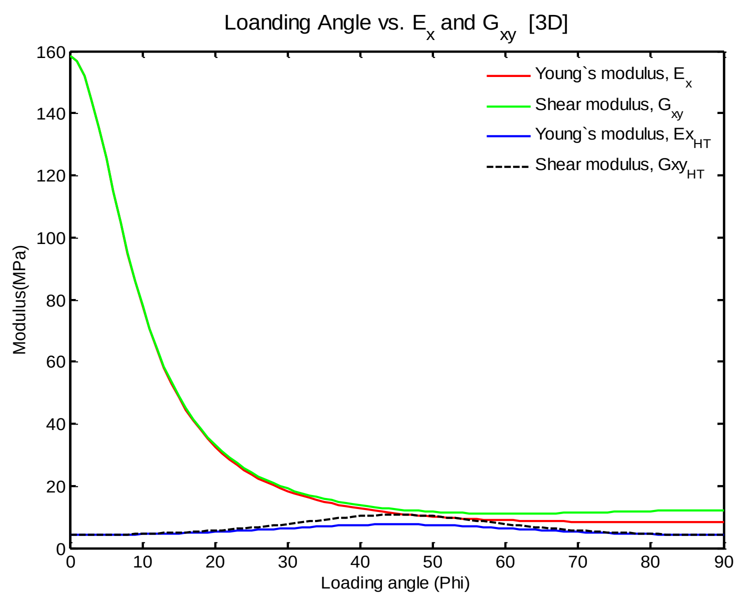


Figure. 3.7: Young’s modulus E_x and the shear modulus G_{xy} for 3D composite structure.

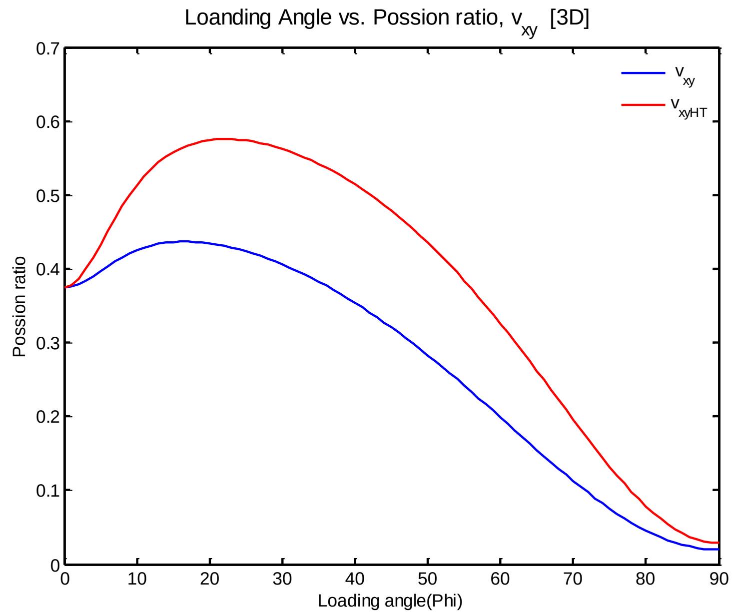


Figure. 3.8: Variation with loading angle Φ of the Poisson ratio υ_{xy} for laminae of ACL at f= 0.5.

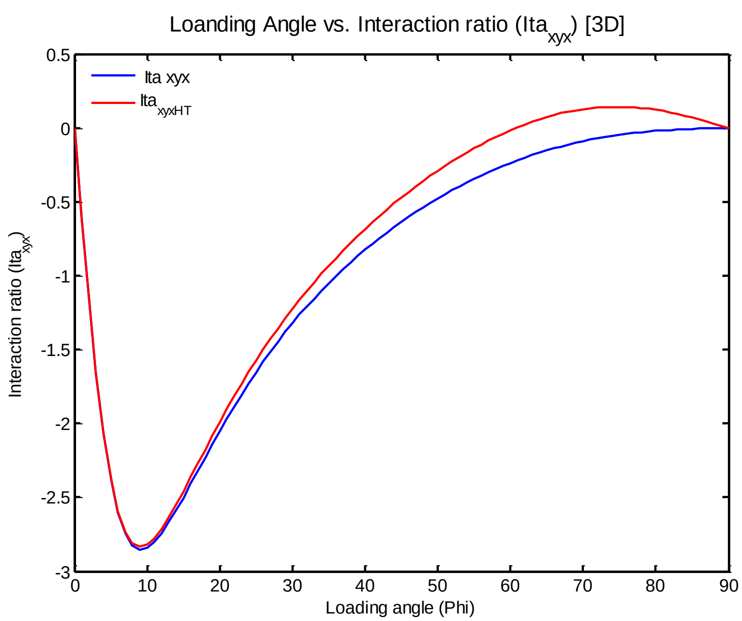


Figure. 3.9. Variation with loading angle Φ of the Poisson ratio υ_{xy} for laminae of ACL at f= 0.5

3.3 The Eshelby Method

Single Ellipsoidal Elastic Model

Probably the single most referenced work in the extensive and rapidly growing literature on elastic composites is Eshelby's Formula for a Single Ellipsoidal Elastic [9] on the response of a single ellipsoidal elastic inclusion in an elastic whole space to a strain imposed at infinity. Eshelby found that a uniform strain at infinity results in a uniform strain within the ellipsoidal inclusion. By using the material properties of collagen fibers and ground matrix, the stiffness tensor C_c was calculated. The Eshelby tensors for the matrix S_m and the fiber S_f were computed as well. Results are presented in 6 by 6 matrices.

Fig. 3.10 shows quantified variations between Rule of Mixture method, HalpinTsia treatment and Eshelby method.

$$C_c(V_f) := \left[\left[C_m^{-1} - V_f\cdot\left[(C_f - C_m)\cdot\left[S - V_f\cdot(S - I)\right] + C_m\right]^{-1}\cdot(C_f - C_m)\cdot C_m^{-1}\right]\right]^{-1}$$

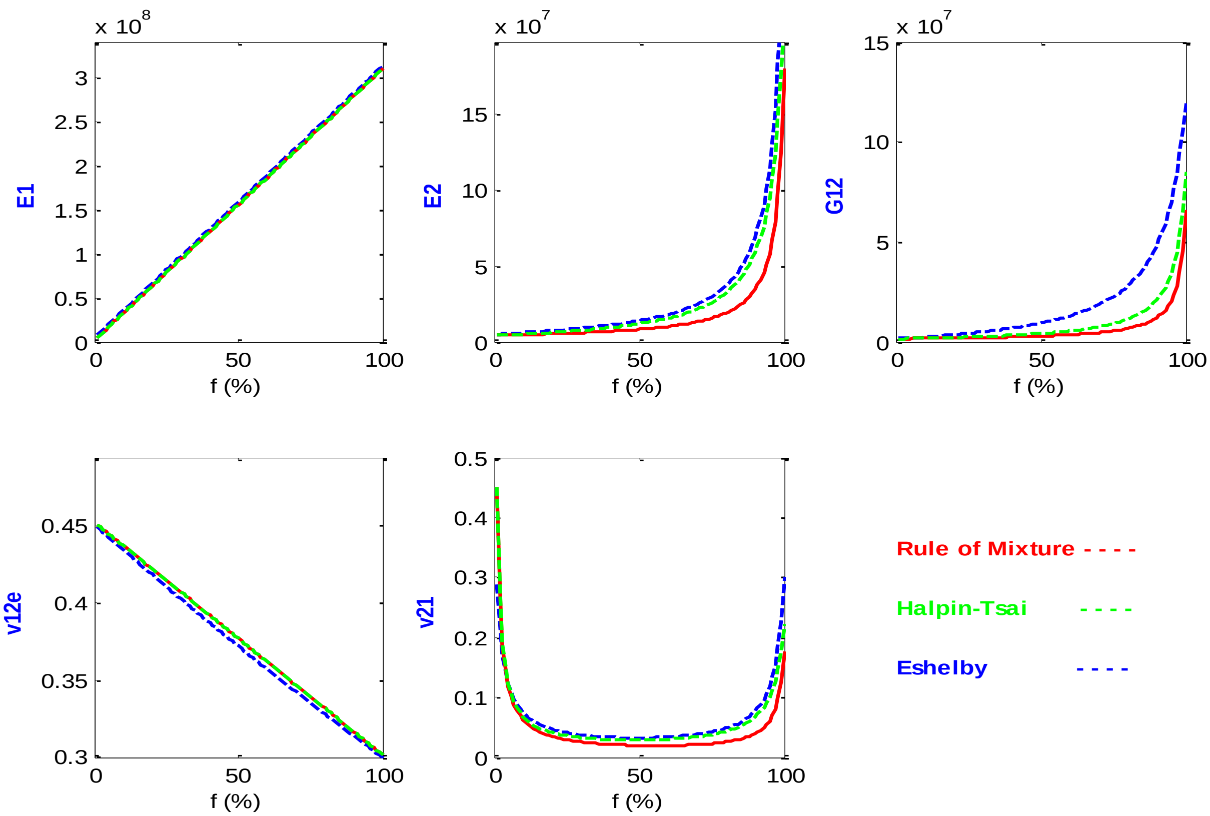


Figure.3.10 shows quantified variations between Rule of Mixture, Halpin-Tsia treatment and Eshelby methods.

3.4. Thermal Ellipsoidal Elastic Model

Since the behavior of composite materials is often sensitive to changes intemperature, analysis of stresses due to change in the temperature is important in order to predicate the thermal expansion coefficient (α_c). These stresses will have associated strains and the net effect of these on the length of a composite that can be calculated or estimated in any given direction. In ACL ligaments there are two main constituents and each has different thermal expansion coefficients. The collagen fibers axial and transverse coefficients are presented in Table 2 and ground matrix have only one coefficient as it thermally behaves the same in both directions [10]. In Fig 3.11, it is important to note that these values are based on elastic behavior and since the ACL ligament is not likely to undergo substantial temperature changes, the model can give reasonable predictions.The core of this technique involves calculating the volume-averaged stresses in both constituents as a result of an imposed misfit strain, so that it is readily adapted for the prediction of expansivities [11].

Table 2. thermal expansion coefficients for the ground matrix and collagen fibers

	Collagen fibers	Ground matrix
Axial	$\alpha_{f,a} = 18 \pm 6 \times 10^{-6}\ K^{-1}$	$\alpha_{m,a} = 50.5 \times 10^{-5}\ K^{-1}$
Transverse	$\alpha_{f,t} = 30 \pm 4 \times 10^{-6}\ K^{-1}$	$\alpha_{m,t} = 50.5 \times 10^{-5}\ K^{-1}$

$$\alpha_c(V_f) := \alpha_m - V_f \times \left[(C_f - C_m) \times [S - V_f \times (S - I)] + C_m\right]^{-1} \times C_f \times (\alpha_f - \alpha_m)$$

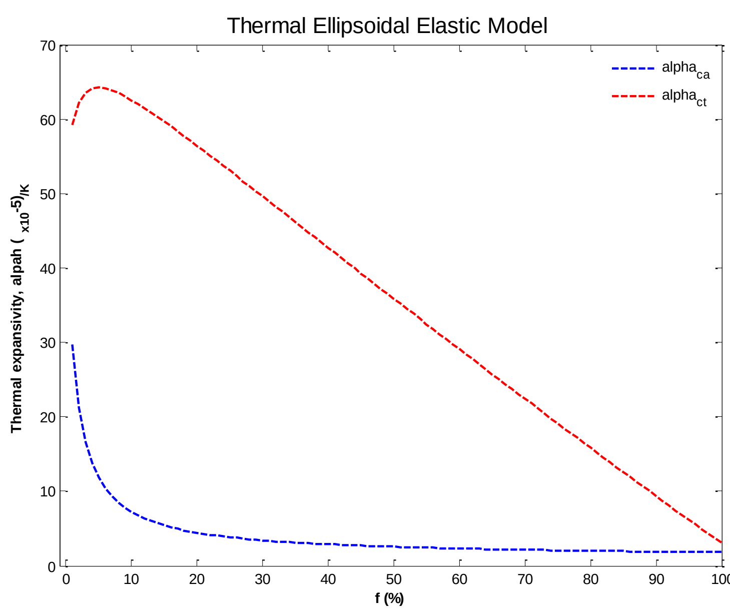


Figure.3.11 Predicted dependence of thermal expansivity on reinforcement contentfor collagen fibers and matrix, according to the force balance and Eshelby models. The curveswere obtained using the property data in Table 2.

4. Conclusion

The orientation of the collagen fibers plays an important role in prescribing the behavior of ACL ligaments. Using Rule of Mixture, HalpinTsia treatment, and Eshelby methods, the behavior of soft tissue like ligaments can be understood. In line with previous studies, when a composite is modeled as a transversely isotropic material, 2D and 3D results are the same [12]. During the modeling of a composite, the kinematics of the ACL ligament varies with the variation in the loading angle Φ. Finally, the predicted thermal expansion coefficient(αc) in axial and transverse of long collagen fibers is based on elastic behavior, therefore the associated internal stresses may become large if the temperature changes are substantial, thus the matrix is likely to undergo plastic flow, or creep. This study may offer a better understanding on how the ACL ligaments elasticity behaves when modeled as a composite comprised of the ground matrix and unidirectional continuous collagen fibers. It may also provide a platform for analysis and visualization of mechanical properties of other human soft tissue when modeled as a composite for bioengineering researchers and students.

5. References

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