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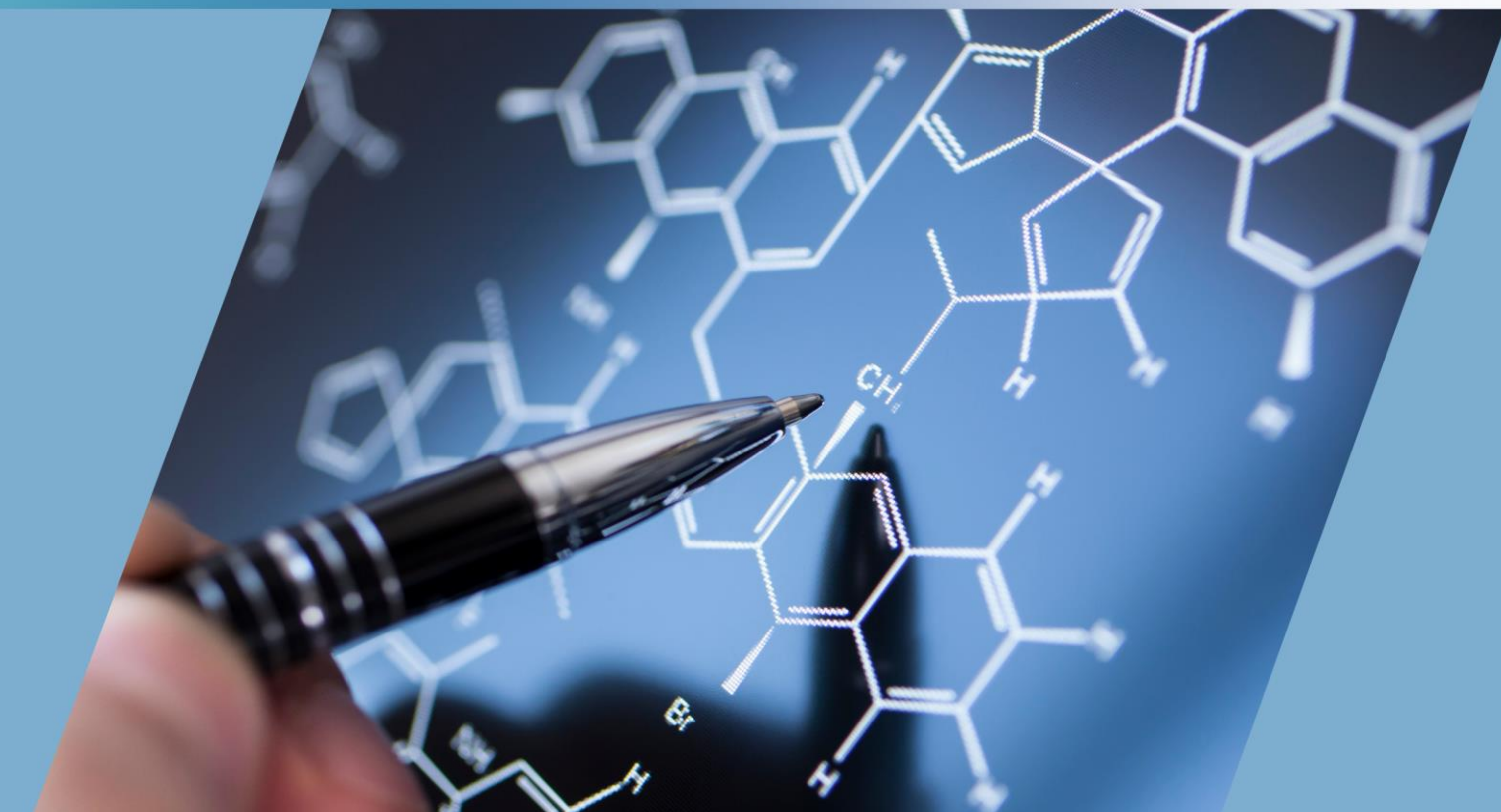

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On Some of Classes of $p -$ Valent $\beta -$ Uniformly Functions

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A B S T R A C T

We focus on the properties of some famous analytical functions. We introduce the classes of *p*- Valent β-uniformly Starlike functions of order α and *p* – Valent β-uniformly Convex functions of order α We come out with new characterization theorems and closure theorems for functions belonging to these classes. Also, we gain radius of p -Valent convexity for functions belonging to the class *p*-valent β-uniformly Convex functions of order α. We insert some notes to explain the evidence of our work.

1 Introduction

The class of analytic functions and p -valent functions in the open deleted unit disk $\mathbb{U} = \{z \in \mathbb{C}: 0 < |z| < 1\}$ has the form:

$$f(z) = z^{p} + \sum_{k=1}^{\infty} a_{p+k} z^{p+k} (p \in \mathbb{N}, \mathbb{N} = \{1,2, \dots\}), \qquad (1)$$

represented by $\mathcal{A}(p)$.

We have some notes:

Note 1: $\mathcal{A}(p) = \mathcal{A}(1)$.

Note 2: If the function $f(z) \in \mathcal{A}(p)$ satisfies the following conditions should be p −valent starlike of order α:

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \; (0 \le \alpha < p, p \in \mathbb{N}; z \in \mathbb{U}). \qquad (2)$$

We denote the class of $p -$ valent starlike functions of order α by $\mathcal{S}_{p}(\alpha)$.

Note 3: If the function $f(z) \in \mathcal{A}(p)$ satisfies the following conditions, it is called α-order *p*-valent convexity:

$$Re\left\{1 + \frac{z f''(z)}{f'(z)}\right\} > \alpha \; (0 \le \alpha < p, p \in \mathbb{N}; z \in \mathbb{U}) \quad (3)$$

We denote the class of p −valent convex functions of order α by $\mathcal{K}_{p}(\alpha)$.

The classes $\mathcal{S}_{p}(\alpha)$ and $\mathcal{K}_{p}(\alpha)$ were investigated by (Patil and Thakare, 2011) and (Owa, 1985). Further from (2) and (3), we can see that

$$f(z) \in \mathcal{K}_{p}(\alpha) \Leftrightarrow \frac{z f'(z)}{p} \in \mathcal{S}_{p}(\alpha) \; (0 \le \alpha < p, p \in \mathbb{N}).$$

For functions $f(z) \in \mathcal{A}$ and $\beta \ge 0$, (Kanas and Wisniowska, 1999& 2000) defined the classes $\beta - \mathcal{UCV}$ and $\beta - \mathcal{ST}$ of β −unifomly convex and β −uniformly star like functions, respectively, see (Kanas, 1999) and (Kanas and Srivastava, 2000).

(Marouf, 2009) with $l = 2, m = 1, \alpha_1 = \beta_1$ *and* $\alpha_2 =$ 1] and (Salim et al., 2011), with $n = 2$] checked the classes $\beta - \mathcal{S}_{p}(\alpha)$ and $\beta - \mathcal{K}_{p}(\alpha)$ for $f(z) \in \mathcal{A}(p)$. p −valent β −uniformly star like and $p -$ valent β −uniformly convex of order α $(0 \le \alpha < p)$ are as follows:

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Definition1 (Marouf, 2009 & Salim et al., 2011). *For* $0 \le \alpha < \wp, \beta \ge 0, \wp \in \mathbb{N}$ *and* $z \in \mathbb{U}$ *, let* $\beta - \mathcal{S}_{\wp}(\alpha)$ *be the class of* $f(z) \in \mathcal{A}(\wp)$ *which satisfy:*

$$Re\left\{\frac{z\,f'(z)}{f(z)} - \alpha\right\} > \beta\left|\frac{z\,f'(z)}{f(z)} - \wp\right|. \tag{4}$$

Definition 2 (Marouf, 2009 & Salim et al., 2011). *For* $0 \le \alpha < \wp, \beta \ge 0, \wp \in \mathbb{N}$ *and* $z \in \mathbb{U}$ *, let* $\beta - \mathcal{K}_{\wp}(\alpha)$ *be the class of* $f(z) \in \mathcal{A}(\wp)$ *which satisfy:*

$$Re\left\{1 + \frac{z\,f''(z)}{f'(z)} - \alpha\right\} > \beta\left|\frac{z\,f''(z)}{f'(z)} - \wp\right|. \tag{5}$$

From (4) and (5) we get

$$f(z) \in \beta - \mathcal{K}_{\wp}(\alpha) \Leftrightarrow \frac{z\,f'(z)}{\wp} \in \beta - \mathcal{S}_{\wp}(\alpha).$$

We have noticed: for $\beta = 1$ the above classes were investigated by (Al-Kharsani and Al-Hajiry, 2008).
By taking $\beta = 0$ in (4) and (5), we obtain classes $\mathcal{S}_{\wp}(\alpha)$ and $\mathcal{K}_{\wp}(\alpha)$ of $\wp -$ valence starlike functions of order $(0 \le \alpha < \wp)$ and $\wp -$ valence convex functions of order $\alpha\,(0 \le \alpha < \wp)$ which were introduced and studied by (Patil and Thakare, 1983) and (Owa, 1983).

Denote by $\mathcal{T}(\wp)$ the subclass of $\mathcal{A}(\wp)$ contains functions of the form:

$$f(z) = z^{\wp} - \sum_{k=1}^{\infty} a_{\wp+k} z^{\wp+k}\ \left(a_{\wp+k} \ge 0, \wp \in \mathbb{N}\right), \tag{6}$$

and define two further classes:

$$\mathcal{TS}_{\wp}(\alpha,\beta) = \beta - \mathcal{S}_{\wp}(\alpha) \cap \mathcal{T}(\wp)$$

and

$$\mathcal{TK}_{\wp}(\alpha,\beta) = \beta - \mathcal{K}_{\wp}(\alpha) \cap \mathcal{T}(\wp).$$

In this paper to prove the main results, we need the next lemmas given by (Marouf ,2009), with $l = 2$, $m = 1, \alpha_1 = \beta_1$ *and* $\alpha_2 = 1$ and (Salim et al.,2011) with $n = 2$.

Lemma1. *See* (Marouf, 2009) and (Salim et al., 2011).
A function $f(z)$ *in (6) belongs to* $\mathcal{TS}_{\wp}(\alpha,\beta)$ *if it satisfies:*

$$\sum_{k=1}^{\infty}[(\wp + k)(1+\beta) - (\alpha + \wp\beta)]a_{\wp+k} \le \wp - \alpha.$$

Lemma2. *See* (Marouf, 2009) and (Salim et al., 2011).
A function $f(z)$ *in (6) belongs to* $\mathcal{TK}_{\wp}(\alpha,\beta)$ *if it satisfies:*

$$\sum_{k=1}^{\infty}\left(\frac{\wp + k}{\wp}\right)[(\wp + k)(1+\beta) - (\alpha + \wp\beta)]a_{\wp+k} \le \wp - \alpha.$$

Notably, a function $f(z)$ in (6) and belongs to $\mathcal{TS}_{\wp}(\alpha,\beta)$. Lemma 1 immediately yields

$$a_{\wp+1} \le \frac{\wp-\alpha}{(1+\beta)+(\wp-\alpha)}, \tag{7}$$

While a function $f(z)$ in (6) which belongs to $\mathcal{TK}_{\wp}(\alpha,\beta)$, Lemma 2 immediately yields

$$a_{\wp+1} \le \frac{\wp(\wp-\alpha)}{[(\wp+1)(1+\beta)-(\wp-\alpha)]}. \tag{8}$$

By taking into account the inequalities (7) and (8), respectively, it seems to be important to introduce two classes, $\mathcal{TS}_{\wp}(\alpha,\beta)$ and $\mathcal{TK}_{\wp}(\alpha,\beta)$ of uniformly $\wp -$valente functions; $\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$ denotes the subclass of $\mathcal{TS}_{\wp}(\alpha,\beta)$ contains of functions of the form

$$f(z) = z^{\wp} - \frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} - \sum_{k=2}^{\infty} a_{\wp+k} z^{\wp+k}, \tag{9}$$

$$a_{\wp+k} \ge 0, \wp \in \mathbb{N}, 0 \le \alpha < \wp, \beta \ge 0, 0 \le \gamma < 1.$$

And $\mathcal{TK}_{\wp,\gamma}(\alpha,\beta)$ denotes the subclass of $\mathcal{TK}_{\wp}(\alpha,\beta)$ consists of functions of the form

$$f(z) = z^{\wp} - \frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} - \sum_{k=2}^{\infty} a_{\wp+k} z^{\wp+k}, \tag{10}$$

$a_{\wp+k} \ge 0,\ \wp \in \mathbb{N}, 0 \le \alpha < \wp,\ \beta \ge 0, 0 \le \gamma < 1.$

We note that:
(i) $\mathcal{TS}_{\wp,\gamma}(\alpha,0) = \mathcal{T}_{\gamma}^{*}(\wp,\alpha,)$ *and* $\mathcal{TK}_{\wp}(\alpha,0) = \mathcal{C}_{\gamma}(\wp,\alpha,)$ (see Aouf et al., 2000).

In addition, one can see (Aouf et al., 2016) and (Alharayzeh, and Ghanim 2022).

2 Characterization Theorems for the Classes $\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$ and $\mathcal{TK}_{\wp,\gamma}(\alpha,\beta)$

Throughout our present paper, we assume that:

$$\wp \in \mathbb{N}, 0 \le \alpha < \wp, 0 \le \gamma < 1 \text{ and } z \in \mathbb{U}.$$

Firstly, we must prove the following theorem.

Theorem1. Suppose that $f(z)$be defined by (9). Then $f(z) \in \mathcal{S}_{\wp,\gamma}(\alpha,\beta)$ if it satisfies:

$$\sum_{k=2}^{\infty}[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]a_{\wp+k} \leq (\wp-\alpha)(1-\gamma). \quad (11)$$

The above result (11) is conclusive for $f(z)$ of the form

$$f(z)=z^{\wp}-\frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]}z^{\wp+k}.$$

Proof.
By setting

$$a_{\wp+1}=\frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]}$$

in Lemma1 and simplifying the inequality (7), we arrived at the assertion (11) of Theorem1. ■

If we set

$$a_{\wp+1}=\frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]}$$

in Lemma 2, we similarly get the next theorem.

Theorem 2. Suppose that $f(z)$ is defined by (*10*). Then $f(z) \in \mathcal{TK}_{\wp}(\alpha,\beta)$ if satisfies:

$$\sum_{k=2}^{\infty}\left(\frac{\wp+k}{\wp}\right)[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]a_{\wp+k} \leq (\wp-\alpha)(1-\gamma).$$

Theorem2 is conclusive for $f(z)$ of the form

$$f(z)=z^{\wp}-\frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\frac{\wp(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]}z^{\wp+k}$$

3 Closure Theorems for the $\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$ and $\mathcal{TK}_{\wp,\gamma}(\alpha,\beta)$

The closure theorem for the $\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$ is given by next theorem.

Theorem 3 . *Let*

$$f_j(z)=z^{\wp}-\frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\sum_{k=2}^{\infty}a_{\wp+kj}z^{\wp+k}$$
$$(a_{\wp+kj}\geq 0; j=1,\dots,m).$$

If $f_j(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)(j=1,\dots,m)$, *and the function* $g(z)$ *given by*

$$g(z)=z^{\wp}-\frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\sum_{k=2}^{\infty}b_{\wp+k}z^{\wp+k}$$

with

$$b_{\wp+k}=\frac{1}{m}\sum_{j=1}^{m}a_{\wp+kj}\geq 0, \quad (12)$$

then $g(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$.

Proof: Because $f_j(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)(j=1,\dots,m)$, from Theorem1, we have

$$\sum_{k=2}^{\infty}[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]a_{\wp+kj} \leq (\wp-\alpha)(1-\gamma)(j=1,\dots,m).$$

Using (12), we get

$$\sum_{k=2}^{\infty}[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]b_{\wp+k}.$$
$$=\sum_{k=2}^{\infty}[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]\left(\frac{1}{m}\sum_{j=1}^{m}a_{\wp+kj}\right)$$
$$=\frac{1}{m}\sum_{j=1}^{m}\left(\sum_{k=2}^{\infty}[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]a_{\wp+kj}\right) \leq (\wp-\alpha)(1-\gamma).$$

Then by Theorem1, $g(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$, which completing the proof■

Theorem 4. *Let*

$$f_j(z)=z^{\wp}-\frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\sum_{k=2}^{\infty}a_{\wp+kj}z^{\wp+k}$$
$$(a_{\wp+kj}\geq 0; j=1,\dots,m)$$

If $f_j(z) \in \mathcal{TK}_{\wp,\gamma}(\alpha,\beta)(j=1,\dots,m)$, then $g(z)$ given by

$$g(z)=z^{\wp}-\frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]}z^{\wp+1}-\sum_{k=2}^{\infty}b_{\wp+k}z^{\wp+k}$$

with $b_{\wp+k}$ defined by *(12)* belongs *to* $\mathcal{TK}_{\wp,\gamma}(\alpha,\beta)$.

Theorem 5. Let

$$f_{\wp+1}(z) = z^{\wp} - \frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} \quad (13)$$

and

$$f_{\wp+k}(z) = z^{\wp} - \frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} - \frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} z^{\wp+k}. \quad (14)$$

Then $f(z) \in \boldsymbol{\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)}$ if and only if it has the form

$$f(z) = \sum_{k=1}^{\infty} c_{\wp+k} f_{\wp+k}(z) \left(c_{\wp+k} \geq 0; \sum_{k=1}^{\infty} c_{\wp+k} = 1 \right). \quad (15)$$

Proof. Suppose that $f(z)$ is given by (15), then from (13) and (14), we find that

$$f(z) = z^{\wp} - \frac{(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} - \sum_{k=2}^{\infty} \frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} c_{\wp+k} z^{\wp+k}$$

where

$$c_{\wp+k} \geq 0, \sum_{k=2}^{\infty} c_{\wp+k} = 1 - c_{\wp+1}.$$

Since

$$\sum_{k=2}^{\infty} [(\wp+k)(1+\beta)-(\alpha+\wp\beta)] \frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} c_{\wp+k} = (\wp-\alpha)(1-\gamma) \sum_{k=2}^{\infty} c_{\wp+k} = (\wp-\alpha)(1-\gamma)\left(1-c_{\wp+1}\right) \leq (\wp-\alpha)(1-\gamma).$$

Then we conclude from Theorem1 that

$f(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$.

Conversely, assume that $f(z)$ defined by (9) belongs to $\boldsymbol{\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)}$.

Then from (11), we have

$$a_{\wp+k} \leq \frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} \; (k \in \mathbb{N} \setminus \{1\}).$$

Setting

$$c_{\wp+k} = \frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} a_{\wp+k} \; (k \in \mathbb{N} \setminus \{1\}),$$

and

$$c_{\wp+1} = 1 - \sum_{k=2}^{\infty} c_{\wp+k}.$$

Here we come with (15). This completes the proof.

Theorem 6. *Let*

$$f_{\wp+1}(z) = z^{\wp} - \frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]} z^{\wp+1}$$

and

$$f_{\wp+k}(z) = z^{\wp} - \frac{\wp(\wp-\alpha)\gamma}{(\wp+1)[(1+\beta)+(\wp-\alpha)]} z^{\wp+1} - \frac{\wp(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} z^{\wp+k}$$

Then $f(z) \in \boldsymbol{\mathcal{KS}_{\wp,\gamma}(\alpha,\beta)}$ if and only if it can be expressed in the form *(15).*

4 The Radius of $\wp-$ valent Convexity for the Class $\mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$.

Here, we will prove the next theorem.

Theorem7 . Let $f(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$, then $f(z)$ *is* $\wp-$ valent convex function of order $\delta(0 \leq \delta < \wp$ *in* $|z| < r_1 = r_1(\wp,\alpha,\beta,\delta,\gamma)$, where $r_1(\wp,\alpha,\beta,\delta,\gamma)$ is the largest value *of* r satisfies:

$$\frac{(\wp+1)(\wp-\alpha)[(1+\beta)+(\wp-\delta)]\gamma}{[(1+\beta)+(\wp-\alpha)]} r + \frac{[(\wp+k)(1+\beta)+(\delta+\wp\beta)](\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]} r^k \leq \wp(\wp-\delta)$$

$$(0 \leq \delta < \wp; \wp \in \mathbb{N}). \quad (16)$$

The result (16) is conclusive for $f_{\wp+k}(z)$ *given by (14).*

Proof. It is sufficient to show for $f(z) \in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$, that

$$\left|1 + \frac{zf''(z)}{f'(z)} - \wp\right| \leq \wp - \delta, |z| < r_1(\wp,\alpha,\beta,\delta,\gamma),$$

where $r_1(\wp,\alpha,\beta,\delta,\gamma)$ is the largest value of r for which the inequality (16) holds true. For $f(z)$ in (9), we have

$$\left|1 + \frac{zf''(z)}{f'(z)} - \wp\right| \leq \frac{\frac{(\wp+1)(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} r + \sum_{k=2}^{\infty} k(\wp+k) a_{\wp+k} r^k}{\wp - \frac{(\wp+1)(\wp-\alpha)\gamma}{[(1+\beta)+(\wp-\alpha)]} r + \sum_{k=2}^{\infty} (\wp+k) a_{\wp+k} r^k},$$

thus

$$\left|1 + \frac{zf''(z)}{f'(z)} - \wp\right| \leq \wp - \delta, |z| < r(\wp,\alpha,\beta,\delta,\gamma),$$

if and only if

$$\frac{(\wp+1)(\wp-\alpha)(\wp+1-\delta)\gamma}{[(1+\beta)+(\wp-\alpha)]}r+$$

$$\sum_{k=2}^{\infty}(\wp+k)(\wp+k-\delta)a_{\wp+k}r^{k}\le \wp(\wp-\delta)(0\le\delta<\wp).$$

Since $f(z)\in \mathcal{TS}_{\wp,\gamma}(\alpha,\beta)$, in view of Theorem1, we may set

$$a_{\wp+k}=\frac{(\wp-\alpha)(1-\gamma)}{[(\wp+k)(1+\beta)-(\alpha+\wp\beta)]}c_{\wp+k}$$

where

$$c_{\wp+k}\ge 0;\sum_{k=2}^{\infty}c_{\wp+k}\le 1.$$

Now, for fixed r, we choose a positive integer number $k_0=k_0(r)$ for which r^k is maximal. Then

$$\sum_{k=2}^{\infty}(\wp+k)(\wp+k-\delta)a_{\wp+k}r^{k}\le \frac{(\wp+k_0)(\wp+k_0-\delta)(\wp-\alpha)(1-\gamma)}{[(\wp+k_0)(1+\beta)-(\alpha+\beta\wp)]}r^{k_0}.$$

Consequently, $f(z)$ is a $\wp-$ valent convex function of the order $\delta(\le\delta<\wp)$ in $|z|<r_1=r_1(\wp,\alpha,\beta,\delta,\gamma)$, provided that

$$\frac{(\wp+1)(\wp-\alpha)(\wp+1-\delta)\gamma}{[(1+\beta)+(\wp-\alpha)]}r+\frac{(\wp+k_0)(\wp+k_0-\delta)(\wp-\alpha)(1-\gamma)}{[(\wp+k_0)(1+\beta)-(\alpha+\beta\wp)]}r^{k_0}$$

5 Conclusion

In this paper, it has been considered some classes of *p*-valent β-uniform analytical functions of order α. By selecting different values for each of the parameters *p*, β, and α and defining new classes of analytic functions, more extent and general results can be obtained for future work.

Conflict of Interest: The author declares that there are no conflicts of interest.

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