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Oscillation of Super-linear second Order Nonlinear Differential Equations with Damping Term

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A B S T R A C T

The study of differential equations has been the object of many researchers over the last decades. Different approaches and various techniques have been adopted to investigate the qualitative properties of their solutions. Recently and driven by their widespread applications, the investigation of differential equations of second order has drawn significant attention. The oscillation of solutions has been the main features that have attracted consideration. Therefore, it has been intended to use the Riccati Transformation Technique for obtaining several new oscillation criteria for different classes of nonlinear differential equations of the second order with a damping term. Oscillatory behavior has taken into account through this study of solutions of some differential equations. Comparisons between our results and the previously known results have presented. The relevance of our theorems has been clear due to carefully selected examples. As a conclusion, our aim is to provide some results to improve and/or extend some of well-known results in the literature.

1 Introduction

This paper concerned with oscillation of the solution to the damped ordinary differential equation of the form:

$$(r(t)\psi(x(t))x'(t))' + h(t)x'(t) + q(t)g(x(t)) = H(t, x(t), x'(t)), \quad (1)$$

where r, ψ, h and q are continuous functions on the interval $[t_0, \infty), t_0 \geq 0$ $r(t)$ is a positive function ,ψ is continuous function on the real line $\mathbb{R}$, with $\psi(x) > 0, \forall x \in \mathbb{R}$ and g is continuously differentiable function on the real line $\mathbb{R}$ except possible at *0* with $xg(x) > 0$ and $g'(x) \geq k > 0$for all $x \neq 0$. $H$is a continuous function on $[t_0, \infty) \times \mathbb{R}^2$ with $\frac{H(t,x(t)x'(t))}{g(x(t))} \leq p(t)$ for all $t\epsilon[t_0, \infty)$

Throughout this study, our attention is only to the solutions of the differential equation (1) that exist on some ray$[t_x, \infty)$ where t_x may depend on the particular solution.

In the past decades, the problems regarding the study of oscillation criteria of differential equations with damping have become an important area of research because such equations arise in many real life problems; see the research papers (Ayanlar & Tiryaki, 2000, Elabbasy & Elhaddad, 2007, Kirane & Rogovchenko, 2001, Mustafa et al., 2004, Nagabuchi &Yamamoto, 1988, Naito, 1974, S. & Rogovchenko, 2003, Rogovchenko &Tuncay, 2007, Rogovchenko &Tuncay, 2008, Rogovchenko & Tuncay, 2009, Saker et al.,2003, Tunc & Avci, 2012, Wang & Song, 2013, Xiaoling & Chenghui, 2013, Zhang & Song, 2011 and Zheng, 2006) and the references cited therein. In particular, second order-damped differential equations are used in the study of vehicle noise, vibration and harshness of vehicles (NVH). In what follows, we present the background details that motivate the contents of this paper.

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(Elabbasy *et al*. 2005, Lu & Meng, 2007, and Rogovchenko &Tuncay, 2008), established some new oscillation conditions of Kamanev and Philos type for the Eq. (1) with $\psi(x(t)) = 1, H(t, x(t), x^{'}(t)) = 0$and they did not sign conditions on$h(t), q(t)$. Rogovchenko & Tuncay, (2009) considered Eq. (1) with $H(t, x(t), x^{'}(t)) = 0$and Berkan (2008), continued the investigation of new oscillation of Eq. (1) but he put $H(t, x(t), x^{'}(t)) = H(t)$.
Zhang & Song, (2011) considered equation (1) when replaced explicit function $q(t)g(x(t))$ by implicit function $Q(t, x)$ they obtained certain necessary criteria for oscillation of Eq. (1).

Wang & Song, (2013), established certain oscillation standards for Eq. (1), with $H(t, x(t), x^{'}(t)) = 0, \frac{g(x(t))}{x(t)} \geq k > 0$ for $x \neq 0$.

Xiaoling & Chenghui, (2013), established an important extension of the celebrated oscillation criteria for (1), they studied it with $\psi(x(t)) = 1, g(x(t)) = g(x(\tau(t)))$ and$H(t, x(t), x^{'}(t)) = 0$.

In the same way, we localize generalize Reccati technique to derive new oscillation conditions for Eq. (1). Our results are more general than the previous results. Precisely chosen examples are provided to demonstrate the influence of impulses on the oscillatory actions of all solutions in this class.

2 Definitions

Definition 1. A solution $x(t)$ of Eq. (1) is oscillatory if it has arbitrarily large zeros; otherwise, we call it non-oscillatory. The Eq. (1) is oscillatory if all of its solutions oscillate.

Definition 2. Equation (1) is said to be super-linear if

$0 < \int_{\pm\varepsilon}^{\pm\infty} \frac{du}{g(u)} < \infty$for every all $\varepsilon > 0$

3 Oscillation Results

In this section, I introduce some theorems that include new conditions for ensuring the oscillation of solutions equation (1).

Theorem 1: Suppose that

(1) $c_1 \leq \psi(x(t)) \leq c_2$ for all $x\epsilon\mathbb{R}$ and
(2) $h(t) \leq 0$for $t \geq t_0$hold.

Let ρ be a continuously differentiable positive function over$[T, \infty)$ such that $\rho^{'}(t) \geq 0$

over $[T, \infty)$; $(\rho'(t)r(t))' \leq 0$and such that
(3) $\lim_{t\to\infty} \int_{T_0}^{t} \frac{1}{\rho(s)r(s)} ds = \infty,$
(4)
$\lim_{t\to\infty} \int_{T_0}^{\infty} R(s)ds = \infty; R(s) = \rho(s)[q(s) - p(s)]$
$-\frac{1}{4A} \frac{(\rho^{'}(s))^2}{\rho(s)} r(s),$
A is constant.
Then, every solution of equation (1) is oscillatory.

Proof: Without loss of generality, we may hypothesize that there exists a solution $x(t)$ of equation (1) such that

$$x(t) > 0 on [T, \infty), for some T \geq t_0 \geq 0.$$

Define

$$\omega(t) = \frac{r(t)\psi(x(t))x^{'}(t)}{g(x(t))}, t \geq T \tag{1}$$

For all $t \geq T_0$ then differentiating Equality (1) and using Eq. (1), we obtain

$$\left(\frac{r(t)\psi(x(t))x^{'}(t)}{g(x(t))}\right)^{'} \leq \frac{H(t, x^{'}(t), x(t))}{g(x(t))} - q(t) - \frac{h(t)x^{'}(t)}{g(x(t))} - \frac{r(t)\psi(x(t))x^{'}(t)g^{'}(x(t))x^{'}(t)}{g^2(x(t))}$$

Since $g^{'}(x(t)) \geq k$and using the condition (1) we have

$$\left(\frac{r(t)\psi(x(t))x^{'}(t)}{g(x(t))}\right)^{'} \leq -[q(t) - p(t)] - \frac{h(t)x^{'}(t)}{g(x(t))} - \frac{kr^2(t)\psi^2(x(t))(x^{'}(t))^6}{c_2 g^2(x(t))}, t \geq T \tag{2}$$

Multiplying the inequality (2) by $\rho(t)$ and integrate form T to t we obtain

$$\frac{\rho(t)r(t)\psi(x(t))x^{'}(t)}{g(x(t))} \leq C_T - \int_T^t \rho(s)[q(s) - p(s)]\, ds - \int_T^t \frac{\rho(s)h(s)x^{'}(s)}{g(x(s))} ds + \int_T^t \left[\rho^{'}(s)\omega(s) - A\frac{\rho(s)}{r(s)}\omega^2(s)\right] ds \tag{3}$$

Where $C_T = \frac{\rho(T)r(T)\psi(x(T))x^{'}(T)}{g(x(T))}$.

By using complement square, the inequality presented by (3) can be written as

$$\frac{\rho(t)r(t)\psi(x(t))x^{'}(t)}{g(x(t))} \leq C_T - \int_T^t \rho(s)[q(s)-p(s)]\,ds - \int_T^t \frac{\rho(s)h(s)x^{'}(s)}{g(x(s))}\,ds$$

$$+\int_T^t \left[A\frac{\rho(s)}{r(s)}\left(W^2(s) - \left(\frac{\rho^{'}(s)r(s)}{2A\rho(s)}\right)^2\right)\right] \qquad (4)$$

By the Bonnet theorem, for a fixed $\varepsilon_t \in [T,t]$ such that

$$-\int_T^t \frac{\rho(s)h(s)x^{'}(s)}{g(x(s))}\,ds = -\rho(T)h(T)\int_T^{\varepsilon_t} \frac{x^{'}(s)}{g(x(s))}\,ds$$

Since$(-\rho(t)h(t)) \geq 0$ and the equation (1) is super-linear, we have

$$-\infty < \int_T^t -\rho(s)h(s)\frac{x^{'}(s)}{g(x(s))}\,ds \leq B, \qquad (5)$$

where $B = -\rho(T)h(T)\int_{x(T)}^{\infty}\frac{du}{g(u)}$.

By (5) and the condition (4), (3) become

$$\frac{\rho(t)r(t)\psi(x(t))x^{'}(t)}{g(x(t))} \leq C_T + B_1 - \int_T^t R(s)\,ds$$

By the condition (4), we have

$$\lim_{t\to\infty}\frac{\rho(t)r(t)\psi(x(t))x^{'}(t)}{g(x(t))} = -\infty.$$

Thus, there exists $T_1 \geq T$such that $x^{'}(t) < 0$for all $t \geq T_1$ The condition (4) also implies that there exists $T_2 \geq T_1$such that

$$\int_{T_1}^{T_2}\rho(s)(q(s)-p(s))\,ds = 0 \text{and}$$
$$\int_{T_2}^t \rho(s)(q(s)-p(s))\,ds \geq 0 \quad \text{for } t \geq T_2$$

Multiplying equation (1) by$\rho(t)$, from the definitions of the functions and condition (2), we get

$$\rho(t)(r(t)\psi(x(t))x^{'}(t))^{'} + \rho(t)h(t)x^{'}(t)$$
$$+\rho(t)q(t)g(x(t)) = \rho(t)H(t,x(t),x^{'}(t))$$
$$\rho(t)(r(t)\psi(x(t))x^{'}(t))^{'} + \rho(t)g(x(t))q(t)$$
$$\leq \rho(t)g\big(x(t)\big)p(t), t \geq T_2. \qquad (6)$$

Integrate the inequality (6) from T_2 to t we obtain

$$\rho(t)r(t)\psi(x(t))x^{'}(t) \leq \rho(T_2)r(T_2)\psi(x(T_2))x^{'}(T_2) + \int_{T_2}^t \rho^{'}(s)r(s)\psi(x(s))x^{'}(s)\,ds$$
$$-g(x(t))\int_{T_2}^t \rho(s)(q(s)-p(s))\,ds$$
$$+\int_{T_2}^t g^{'}(x(s))x^{'}(s)\int_{T_2}^s \rho(u)(q(u)-p(u))du ds$$

By the condition (1) and the Bonnet's theorem, for $t \geq T_2$there exists $\gamma_t \in [T_2,t]$ such that

$$c_2\rho(t)r(t)x^{'}(t) \leq \rho(T_2)r(T_2)\psi(x(T_2))x^{'}(T_2) + c_1\rho^{'}(T_2)r(T_2)[x(\gamma_t) - x(T_2)]$$
$$-g(x(t))\int_{T_2}^t \rho(s)(q(s)-p(s))\,ds$$
$$+\int_{T_2}^t g^{'}(x(s))x^{'}(s)\int_{T_2}^s \rho(u)(q(u)-p(u))\,du ds, t \geq T_2$$

Thus

$$c_2\rho(t)r(t)x^{'}(t) \leq \rho(T_2)r(T_2)\psi(x(T_2))x^{'}(T_2), t \geq T_2$$

Dividing the last inequality by$\rho(t)r(t)$, integrate from $T_2$to t and the condition (3), we have
$c_2x(t) \leq c_2x(T_2)$
$+\rho(T_2)r(T_2)\psi\big(x(T_2)\big)x^{'}(T_2)\int_{T_2}^t \frac{ds}{\rho(s)r(s)} \to -\infty$
$as\ t \to \infty$, that is a inconsistency to the fact that $x(t) > 0$for $t \geq T$. This complete the proof.

Theorem 2: Suppose that the condition (1) hold, and

(5) $\int_T^{\infty}\frac{ds}{r(s)} \leq k_1, k_1 > 0$

(6) $\int_{\pm\varepsilon}^{\pm\infty}\frac{\psi(u)du}{g(u)} < \infty for all \varepsilon > 0.$

Furthermore, suppose that there exists a positive continuous differentiable function ρ on the interval $[t_0,\infty)$ with $\rho(t)$ is a non-decreasing function on the interval$[t_0,\infty)$such that

(7)$\lim_{t\to\infty} sup \int_T^t \frac{1}{r(s)\rho(s)}\int_T^s \rho(u)\left[q(u) - p(u) - \frac{h^2(u)}{4c_1kr(u)}\right]du ds = \infty,$

where $\rho: [t_0,\infty) \to (0,\infty)$.

Thus, each solution of super-linear equation (1) is oscillatory.

Proof: Without loss of generality, we can suppose that there exists a solution $x(t)$of equation (1) such that $x(t) > 0 on [T,\infty) for some T \geq t_0 \geq 0$. Define

$$\omega(t) = \frac{\rho(t)r(t)\psi(x(t))x'(t)}{g(x(t))}, t \geq T$$

This and by the condition (1) and Eq. (1), we have

$$\omega'(t) \leq \rho(t)p(t) - \frac{\rho(t)h(t)x'(t)}{g(x(t))} - \rho(t)q(t) + \frac{\rho'(t)}{\rho(t)}\omega(t) - \frac{c_1 k\rho(t)r(t)(x'(t))^2}{g^2(x(t))}, t \geq T$$

Thus for $t \geq T$, we have

$$\rho(t)\left(\frac{\omega(t)}{\rho(t)}\right)' \leq \rho(t)p(t) - \rho(t)q(t) - \frac{\rho(t)h(t)x'(t)}{g(x(t))} - \frac{c_1 k\rho(t)r(t)(x'(t))^2}{g^2(x(t))}, t \geq T$$

$$\rho(t)[q(t) - p(t)] \leq -\rho(t)\left(\frac{\omega(t)}{\rho(t)}\right)' - \frac{\rho(t)h(t)x'(t)}{g\big(x(t)\big)} - \frac{c_1 k\rho(t)r(t)(x'(t))^2}{g^2(x(t))}, t \geq T$$

Integrate from T to t, we obtain

$$\int_T^t \rho(s)[q(s) - p(s)]\,ds \leq \int_T^t -\rho(s)\left(\frac{\omega(s)}{\rho(s)}\right)' ds - \int_T^t \frac{\rho(s)h(s)x'(s)}{g(x(s))}ds - \int_T^t \frac{c_1 k\rho(s)r(s)(x'(s))^2}{g^2\big(x(s)\big)}ds, \quad t \geq T$$

$$-\int_T^t \frac{\rho(s)h(s)}{g(x(s))} + \frac{c_1 k\rho(s)r(s)(x'(s))^2}{g^2(x(s))}ds = -\int_T^t \left[\sqrt{c_1 k\rho(s)r(s)}\frac{x'(s)}{g(x(s))} + \frac{1}{2}\sqrt{\frac{\rho(s)}{c_1 kr(s)}}h(s)\right]^2 ds + \frac{1}{4kc_1}\int_T^t \frac{\rho(s)h^2(s)}{r(s)}ds \leq \frac{1}{4kc_1}\int_T^t \frac{\rho(s)h^2(s)}{r(s)}ds$$

By the Bonnet's theorem, since $\rho(t)$ is a non-decreasing function on the interval$[t_0,\infty)$, there exists $T_1 \in [T,t]$such that

$$\begin{aligned} -\int_T^t \rho(s)\left(\frac{\omega(s)}{\rho(s)}\right)' &= -\rho(t)\int_{T_1}^t \left(\frac{\omega(s)}{\rho(s)}\right)' ds \\ &= -\rho(t)\int_{T_1}^t \left(\frac{d}{ds}\frac{\omega(s)}{\rho(s)}\right)ds \\ &= -\rho(t)\frac{\omega(t)}{\rho(t)} + \rho(t)\frac{\omega(T)_1}{\rho(T_1)} \end{aligned} \tag{7}$$

From the inequalities (7) and (5) in the inequality (4), we have

$$\int_T^t \rho(s)\left[q(s) - p(s) - \frac{h^2(s)}{4c_1 kr(s)}\right]ds \leq -\rho(t)\frac{\omega(t)}{\rho(t)} + \rho(t)\frac{\omega(T)_1}{\rho(T_1)}.$$

$$\int_T^t \rho(s)\left[q(s) - p(s) - \frac{h^2(s)}{4c_1 kr(s)}\right]ds \leq -\omega(t) + \rho(t)\frac{\omega(T)_1}{\rho(T_1)}.$$

$$\int_T^t \rho(s)\left[q(s) - p(s) - \frac{h^2(s)}{4c_1 kr(s)}\right]ds \leq -\omega(t)$$

$+\rho(t)\frac{\omega(T)_1}{\rho(T_1)}$. Integrating the last inequality divided by $\rho(t)r(t)$ from T to t, taking the limit superior on both sides and by conditions (5) and (6), we have

$$\limsup_{t\to\infty}\int_T^t \frac{1}{r(s)\rho(s)}\int_T^s \rho(u)\left[q(u) - p(u) - \frac{h^2(s)}{4ckr(s)}\right]duds$$

$$\leq \limsup_{t\to\infty}\left\{\left(\frac{\omega(T_1)}{\rho(T_1)}\right)\int_T^t \frac{ds}{r(s)} - \int_{x(T)}^{x(t)}\frac{\psi(u)du}{g(u)}\right\}$$

$$< \infty \quad as t \to \infty,$$

which contradicts to the condition (7). Hence, the proof is completed.

4 Discussion

A set of new oscillation conditions are stated and proved which extend and improve previous oscillation criteria and cover the cases which are not covered by known results. Some of illustrative examples are provided to show the applications of the oscillation criteria and the comparisons between our results and previous results in the literature.

Example1: Consider the following differential equation

$$\left(\frac{1}{t}\left(\frac{x^6(t)+2}{x^6(t)+1}\right)x'(t)\right)' - t^2x'(t) + \left(t+\frac{\sin t}{t}\right)x^5(t)$$

$$= \frac{2x^{12}(t)\sin t\cos(x'(t)+1)}{(x^7+1)t^3}, t \geq \frac{\pi}{2}$$

Here,

$r(t) = \frac{1}{t}, h(t) = -t^2, q(t) = t + \frac{\sin t}{t}, g(x) = x^5,$
$H(t,x(t),x'(t)) = \frac{2x^{12}(t)\sin t\cos(x'+1)}{(x^7+1)t^3},$

for all$x \neq 0$and $t > 0$.

$$\frac{H(t,x(t),x'(t))}{g(x(t))} = \frac{2x^{12}(t)\sin t\cos(x'+1)}{(x^7+1)t^3} \times \frac{1}{x^5(t)}$$
$$\leq \frac{2}{t^3} = p(t)$$

for all$x \neq 0$and $t > 0$. $\psi(x) = \frac{x^6+2}{x^6+1}$ and

$1 \leq \psi(x) \leq 2$ for all $x \in \mathbb{R}$,
$\rho(t) = t, \rho'(t)r(t) = \frac{1}{t} > 0, (\rho(t)h(t))' = -3t^2 < 0$
$and(\rho'(t)r(t))' = \left(\frac{1}{t}\right)' = \frac{-1}{t^2} < 0 forallt > 0.$
So, can note that

$$\int_{t_0}^{\infty}\frac{ds}{\rho(s)r(s)} = \int_{t_0}^{\infty}ds = \infty,$$

$$R(s) = s^2 + \sin s - \frac{2}{s^2} - \frac{1}{4As^2}$$
$$\int_{t_0}^{\infty}R(s)ds = \infty.$$

All conditions of Theorem 1 are satisfied; thus, the given equation is oscillatory.

Example2: Consider the following differential equation

$$\left(\frac{(x^2(t)+2)}{t^4(x^2(t)+1)}x'(t)\right)' + \frac{x'(t)}{t^5} + t^4x^5(t)$$
$$= \frac{x^5(t)\cos(x(t))}{t^9}, t > 0.$$

We note that

$r(t) = \frac{1}{t}, \ \psi(x) = \frac{x^2(t)+2}{x^2(t)+1} > 0 \ and 1 \leq \psi(x) \leq 2,$

for all $x \in \mathbb{R}, h(t) = \frac{1}{t^5}$, and $\frac{H(t,x(t),x'(t))}{g(x(t))} = \frac{\cos(x(t))}{t^9}$
$$\leq \frac{1}{t^9} = p(t) forall \ t > 0 \ and \ x \neq 0.$$

Let $\rho(t) = t^6$such that

$$\limsup_{t\to\infty}\int_T^t\frac{1}{r(s)\rho(s)}\int_T^s\rho(u)\left[q(u)-p(u) - \frac{h^2(u)}{4c_1kr(u)}\right]duds$$
$$= \limsup_{t\to\infty}\int_T^t\frac{1}{s^2}\int_T^s u^6\left[u^4 - \frac{1}{u^9} - \frac{1}{4c_1ku^6}\right]duds$$

$$= \limsup_{t\to\infty}\left[\frac{s^{10}}{110} - \frac{1}{6s^3} + \frac{1}{8c_1kTs^2} + (\frac{T^{11}}{11s} + \frac{1}{2sT^2} + \frac{1}{4c_1ksT})\right]_T^t = \infty.$$

All conditions of Theorem2 are satisfied and hence each solution of the given equation is oscillatory.

Remark1: Theorem1 and Theorem 2 extend and improve results of (Elabbasy & Elzeine, 2011, Remili, 2008, and Results of Xhevair & Elisabeta, 2014).

Remark2: Remili, (2008) has established some oscillation results for Eq. (1) with $\psi(x(t)) = 1, h(t) = 1$, these results required that $r(t) \leq a_1$ and

$$\liminf_{t\to\infty}\int_T^t Z(s)ds > -\lambda, \quad (\lambda > 0)$$

for all large $T; Z(s) = R(s)[q(s) - p(s)]$, which are not required in Theorem 2.

5 Conclusions

To sum up, a set of new oscillation conditions are stated and proved which extend and improve previous oscillation criteria and cover the cases which are not covered by known results. Further, we introduced some illustrative examples. Remarks were also included to show the evidence of our main results.

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References

Atkinson, F. V. (1955). On second order nonlinear oscillations. *Pacific. J. Math.*, (5), 643-647.

Ayanlar, B. and Tiryaki, A. (2000). Oscillation theorems for nonlinear second order differential equations with damping. *Acta Math. Hungar*., (89), 1-13.

Beqiri, X. and Koci, E. (2014). New Oscillation and Non-oscillation Criteria for Second Order Nonlinear Differential Equations. *Int. J. of Pur. and Appl. Math.*,93(2), 155-163.

Bhatia, N. P. (1966). Some oscillation theorems for second order differential equations. *J. Math. Anal. Appl.*, (15), 442-446.

Bihari, I. (1963). An oscillation theorem concerning the half linear differential equation of the second order. *Magyar Tud. Akad. Mat. Kutato Int.Kozl.*, (8), 275-280.

Berkane, A. (2008). Sufficient conditions for the oscillation of solutions to nonlinear second order differential equations. *Electronic J. of Differential Equations*, (3), 1-6.

Elabbasy, E. M. and Elzeine, S. R. (2011). Oscillation theorems concerning nonlinear differential equations of the second order, *Opuscula Math.*, (31), 373-391.

Lee, Chung-Fen and Yeh, Chen-Chin (2007). An oscillation theorem, *Applied Mathematics Letters*, (20), 238-240.

El-abbasy, E. M. Hassan, Taher S. and Saker, Samir H. (2005). Oscillation of second- order non-linear differential equations with a damping term. *Electronic Journal of differential equations vol.*, (76), 1-13.

Elabbasy, E. M. and Elhaddad, W. W. (2007). Oscillation of second order non-linear differential equations with a damping term. *Electronic Journal of Qualitative Theory of Differential Equations*, (25), 1-19.

Fite, W. B. (1918). Concerning the zeros of the solutions of certain differential equations. *Trans. Amer. Math. Soc.*, (19), 341-352.

Grace, S. R. and Lalli, B. (1980). Oscillation theorems second perturbed differential equations. *J. Math. Anal. Appl.*, (77), 205-214.

Greaf, J. R. Rankin, S. M. and Spikes, P. W. (1978). Oscillation theorems for perturbed nonlinear differential equations. *J. Math. Anal. Appl.* (65), 375-390.

Kamenev, I. V. (1978). Integral criterion for oscillation of linear differential equations of second order. *Math. Zametki* (23), 249-251.

Kartsatos, A. G. (1968). On oscillations of nonlinear equations of second order. *J. Math. Anal. Appl.*, (24), 665-668.

Kirane, M. and Rogovchenko, Y. V. (2000). Oscillation results for second order damped differential equations with nonmonotonous nonlinearity. *J. Math. Anal. Appl.* (250), 118-138.

Kirane, M. and Rogovchenko, Y. V. (2001). On oscillation of nonlinear second order differential equation with damping term. *Appl. Math. and Comput*., (117), 177-192.

Lu, F. and Meng, F. (2007). Oscillation theorems for superlinear second order nonlinear damped differential equations. *Appl. Math. and Comput.*, (189), 796-804.

Remili, M. (2008). Oscillation theorem for perturbed nonlinear differential equations. *Int. Math. Forum.*, (11) 513-524.

Mustafa, O. G. Rogovchenko, S. P. and Rogovchenko, Y. V. (2004). On oscillation of nonlinear second order differential equations with damping term. *J. Math. Anal. Appl.* (298), 604-620.

Nagabuchi, Y. and Yamamoto, M. (1988). Some oscillation criteria for second order nonlinear ordinary differential equations with damping. *Proc. Japan. Acad.*, (64), 282-285.

Naito, M. (1974). Oscillation criteria for a second order differential equation with a damping term. *Hiroshima Math. J.* (4), 285-291.

Manojlovic, J. V. (2001). Integral averages and oscillation of second order nonlinear differential equations. *Computers and Math. Applic.*, (41),1521-1534.

Remili, M. (2010). Oscillation criteria for second order nonlinear perturbed differential equations. *Electronic Journal of Qualitative Theory of Differential Equations*, (25), 1-11.

Rogovchenko, S. P. and Rogovchenko, Y. V. (2003). Oscillation of differential equations with damping. *Dyn. Contin. Discrete Impuls. Sys. Ser. A Math. Anal.*, (10), 447-461.

Rogovchenko, Y. V. and Tuncay, F. (2007). Interval oscillation of a second order nonlinear differential equation with a damping term. *Discrete and Continuous Dynamical Systems Supplement*, 883-891.

Rogovchenko, Y. V. and Tuncay, F. (2008). Oscillation criteria for second order nonlinear equations with damping. *Nonlinear Analysis*, (69), 208-221.

Rogovchenko, Y. V. and Tuncay, F. (2009). Oscillation theorems for a class of second order Nonlinear differential equations with damping. *Taiwanese J. of Math.*, (13), 1909-1928.

Saker, H. S. Pang, P. Y. H. and Agrawal, R. P. (2003). Oscillation theorems for second order nonlinear functional differential equations with damping. *Dynamic Sys. Appl.*, (12), 307-322.

Temtek, P. and Tiryaki, A. (2013). Oscillation criteria for a certain second order nonlinear perturbed differential equations. *J. of Inequalities and Appl.*, 524 1-12.

Tunc, E. and Avci, H. (2012). Oscillation criteria for a class of second order nonlinear differential equations with damping. *Bull. of Math. Anal. and Appl.*, 4(2) 40-50.

Wang, X. J. and Song, G. H. (2011). Oscillation criteria for a second order nonlinear damped differential equation. *Inter. J. of Info. and Syst. Scie.*, 7 73-82.

Wang, X.J. and Song, G. H. (2013). Oscillation theorems for a class of nonlinear second order differentia;l equations with damping. *Advances in Pure Mathematics*, (3) 226-233.

Xiaoling, F. L. and Chenghui, Z. (2013). Oscillation of second-order dampeddifferential equations. *Advances in Difference Equations.*, 326 1-11.

Xhevair, B. and Elisabeta, K. (2014). New Oscillation and Non-oscillation Criteria for Secnd Order Nonlinear Differential Equations. *International Journal of Pure and Applied Mathematics.*, 93 (2), 155-163.

Zhang, M. and Song, G. H. (2011) Oscillation theorems for second order nonlinear perturbed differential equations with damping. J. Shanghai Univ. (Engl. Ed.) 15(6) 510-516.

Zheng, Z. (2006). Oscillation criteria for nonlinear second order differential equations with damping. *Acta Math. Hungarica*, (110), 241-252.

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