

Construction of Semi-Parametric Simultaneous Equation Models with An Application

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Abstract

This research aims to construct a semi-parametric simultaneous equation model with internal variables, then reduce, characterize, and estimate the model using kernel estimators and the least squares method in two stages. This provides a statistical framework for addressing the problem of internal homogeneity and nonlinear relationships in semi-parametric simultaneous equation models. Consumption, GDP, money supply, and government spending data for the Iraqi economy were used to implement the model. The results showed the model's superiority in detecting sharp structural changes and tracking sudden shocks. The estimated parameters demonstrated a strong nonlinear behavioral response to consumption and a significant positive impact of fiscal and monetary policy tools. The randomness of errors and the statistical validity of the estimated inference model were confirmed through residual analysis.

Keywords

**Semiparametric
Simultaneous Equation
model,
Model Reduction,
Identification and
Diagnosis,
Semiparametric Models.**

بناء نموذج المعادلات الآنية شبه المعلمية مع التطبيق

مريم جمعه موسى

القسم العلوم المالية والمصرفية – كلية الإمام الكاظم للعلوم الإسلامية الجامعة، بغداد، العراق

الكلمات المفتاحية

الملخص

نماذج المعادلات الآنية شبه المعلمية،
اختزال النماذج،
تشخيص وتحديد النماذج،
النماذج شبه المعلمية.

هدف هذا البحث إلى بناء نموذج معادلات آنية شبه معلمية بمتغيرات داخلية، ثم اختزال النموذج وتوصيفه وتقديره باستخدام مقدرات النواة وطريقة المربعات الصغرى على مرحلتين، وذلك لتقديم إطار إحصائي لمعالجة مشكلة التجانس الداخلي والعلاقات غير الخطية في نماذج المعادلات الآنية شبه المعلمية. وقد استُخدمت بيانات الاستهلاك والناتج المحلي الإجمالي والمعرض النقدي والإنفاق الحكومي للاقتصاد العراقي لتطبيق النموذج. وأظهرت النتائج تفوق النموذج في رصد التغيرات الهيكلية الحادة وتتبع الصدمات المفاجئة. وأظهرت المعلمات المقدرة استجابة سلوكية غير خطية قوية للاستهلاك وتأثيراً إيجابياً كبيراً لأدوات السياسة المالية والنقدية. وقد تم تأكيد عشوائية الأخطاء والصحة الإحصائية لنموذج الاستدلال المقدر من خلال تحليل البواقي.

1.Introduction

Modeling correlated random systems is one of the most important problems in modern econometric research, where the phenomena observed are often such that they interact and feed back simultaneously in a statistically dynamic way, which causes the problem of endogeneity. In such an econometric setup, the factor that explains the dependent variable is correlated with the random term in the regression model, causing a violation of the basic assumptions of the Ordinary Least

Squares (OLS) method which results in the bias and inconsistency of the estimator.

Models of this type, which have been fully linearized in a parametric form, have been introduced and presented in the statistical literature for a long time as a mathematical tool for solving the problem. The selection of the predetermined functional forms, however, can be quite restrictive and may result in large specification error particularly in time series displaying abrupt changes and structural breaks. Nonparametric models, by contrast, have great flexibility in that they

can directly learn the missing functional forms from the data without imposing any a priori restrictions. Mathematically, however, they are faced with the ill-posed inverse problem in an internally homogeneous environment, slow convergence rates of estimators and the so-called "curse of dimensionality."

Thus, This study aimed to construct a semi-parametric simultaneous equation model and reduce the interconnected semi-parametric simultaneous equation system to an algebraic matrix framework with rank and degree conditions. The proposed model was estimated by combining kernel estimators and the two-stage least squares (2SLS) method. To evaluate the empirical efficiency and predictive power of this model and its semi-parametric estimators, real macroeconomic data from Iraq (1990–2022) were used.

Then to validate the semiparametric simultaneous equation model and support statistical inference, the study addresses the following hypothesis testing framework:

1) Structural Parameters Hypothesis:

Null Hypothesis (H_0): The structural parameters of the semiparametric model are statistically insignificant ($\alpha_k = 0, \beta_k = 0$), indicating no significant simultaneous feedback between private consumption and GDP.

Alternative Hypothesis (H_1): The structural parameters are statistically significant ($\alpha_k \neq 0, \beta_k \neq 0$) at the 5% significance level ($p < 0.05$), confirming the validity of structural channels and simultaneous policy transmission.

2) Non-parametric Component Hypothesis:

Null Hypothesis (H_0): The non-parametric component $m(y_{2t})$ follows a strictly linear form ($\alpha_3 = 0$).

Alternative Hypothesis (H_1): The non-parametric component $m(y_{2t})$ exhibits significant non-linear behavior ($\alpha_3 \neq 0$), capturing structural behavioral dynamics.

3) Model Adequacy and Specification Hypothesis:

Null Hypothesis (H_0): The proposed semiparametric simultaneous equation model is inadequate and does not statistically improve the representation of structural relationships compared to traditional linear models.

Alternative Hypothesis (H_1): The proposed semiparametric model exhibits superior goodness-of-fit and statistical adequacy ($p < 0.05$) in capturing structural and simultaneous dynamics, outperforming standard linear specifications.

2 .Literature Review

In order to place the methodology behind semiparametric simultaneous equation models into the context of current econometric theory, it will be necessary to first conduct a survey of some important prior research.

Then the Kim (2000) The author constructed a Pseudo-Adaptive Maximum Likelihood (PAML) estimator for the nonlinear simultaneous equations, which gave your study very flexible semiparametric estimators that do not require the strong distributional assumptions that are necessary for skewed Iraqi monetary variables when it comes to continuous external shocks.

An Rodríguez-Póo et al. (2001) The researchers proposed an additive structure for semiparametric triangular systems, which was useful in solving the curse of dimensionality and structural reduction, and was used to simultaneously estimate structural expenditure functions and consumer behavior under economic choice constraints in Iraq.

Then in 2002 Imbens & Newey was the first to break the additivity assumption between the structural regressors and the error terms, laying a solid theoretical basis for the structural diagnosis and identifications of control function estimators when measuring the asymmetric impacts of oil shock on the Iraqi economy.

And Kelejian and Prucha (2004) Presented methods to estimate systems of spatial equations that are overlapping along cross-sections and the study showed consistent and coherent estimates of parameters in the case of spatial correlation.

Su & Ullah (2008) The authors extended simultaneous equations to a fully nonparametric framework, through local polynomial estimation, which provides a methodologically sound justification for using a "semi-parametric" design that would ease the data reduction task involved in the estimation of supply-demand distortions in the Iraqi market.

And the Gao and Phillips in (2010) revolutionized the structural setup by separating endogenous parametric regressors from nonparametric stochastic trends, directly serving the statistical diagnosis of nonstationary and cointegration when modeling the

simultaneous feedback between Iraqi monetary and fiscal policies. Ando & Zellner (2010) The researchers formulated a hierarchical Bayesian framework for simultaneous system estimation and prediction under severe uncertainty, enhancing your computational discussion when simulating the empirical performance of the model against target macroeconomic growth plans in Iraq amid global energy price volatility.

The study by Akerberg et al. (2014) provided an in-depth theoretical analysis of the efficiency of two-stage semiparametric approaches within the framework of the Generalized Moments Method (GMM), and demonstrated that the 2SLS semiparametric estimators achieve the semi-informed efficiency limit.

Hsiao and Zhou's study (2015) presented statistical conclusions for dynamic real-time equation models of tablet data, and the researchers proved the possibility of identification and estimation in the presence of fixed individual effects.

The study by Chen and Qiu (2016) provided a comprehensive guide to non-parametric and semi-parametric methods for dealing with the problem of internal homogeneity through a detailed discussion of the use of the sieve and penalization methods. Yang (2016) The author expanded the structural architecture by embedding spatial autoregressive components into simultaneous systems, thereby supporting the estimation and identification strategy of your model should the empirical application be extended to capture cross-provincial spatial spillovers within the Iraqi economy. Berry & Haile (2016) This paper resolved critical diagnosis and identification dilemmas in nonseparable simultaneous models through a "residual index structure," ensuring rigorous model specification to address endogeneity and weak instruments within the Iraqi credit and banking sectors.

Wang (2017) The study devised the advanced conditional moment techniques for bias assessment and reduction in limited information dynamic estimators providing you the "best" mathematical solution to your parametric component in the case of short or broken time-series as are Iraqi macroeconomic aggregates. Yang (2017) The dissertation combined the semiparametric Generalized Estimating Equations (GEE) with kernel smoothers for longitudinal data, providing a solid set of bandwidth selection criteria to maximize the precision of these GEE estimators

when applied to banking panels and/or firms from the Iraq Stock Exchange (ISE).

The authors of Pérez-Sánchez et al. (2021) proposed a paradigm change by using entropy minimization to estimate the models instead of the distance-based approach, and flexible algorithmic solutions that can be effective in the presence of high dispersion and structural outliers within the Iraqi financial context.

Ding & Zhou (2025) This modern research project developed a novel and state-of-the-art approach to estimation of time-varying nonlinear time-series using sieve nonparametric estimators, directly moving forward the diagnosis pillar of your thesis to investigate potential sharp regime shifts or structural invariance of the semiparametric parameters during successive geopolitical crises in Iraq.

While the econometric literature has explored combinations of parametric and nonparametric components in models, as noted below, the non-parametric function is typically made to depend on strictly exogenous variables or on deterministic time trends (e.g., Gao & Phillips, 2010).

This study addresses this important methodological lacuna through the development of an innovative semiparametric framework in which the endogenous regressor is directly included in the non parametric mapping. The structural form enables the model to reproduce complex non-linear dynamics and includes multiple feedback loops between the endogenous variables. In addition, the paper derives closed-form, non-iterative and direct algebraic solutions for structural parameter extraction, combined with a control function approach to overcome non-parametric endogeneity, which is not possible with the complex iterative numerical optimization procedures previously proposed in the literature, where non-convergence of the computations is a common issue in small samples.

3-Constructing a Semi-Parameter Simultaneous Equations Model

The research proposal involves constructing a system of simultaneous equations that integrates linear (parametric) and non-parametric unknowns in an interwoven manner. This model differs from traditional models in that it assumes that the interaction between endogenous variables occurs within the non-parametric structure itself. The proposed system is formulated as follows:

$$y_{1t} = \alpha_1 + \alpha_2 z_{1t} + \alpha_3 m(y_{2t}) + \epsilon_{1t} \quad \dots (1)$$

$$y_{2t} = \beta_1 + \beta_2 z_{2t} + \beta_3 y_{1t} + \epsilon_{2t} \quad \dots (2)$$

Where :

y_{1t}, y_{2t} Internal variables.

z_{1t}, z_{2t} External variables.

$m(.)$ Non-parametric function.

The advantage of this model over the previous literature (e.g. Gao & Phillips, 2010 model) is the type of independent variable in the non-parametric portion of the reference model. The non-parametric function is built on an extrinsic variable (Trend), while in Equation 1, the function $m(y_{2t})$ is built on an internal, simultaneous variable, thus increasing the accuracy of the simulation. The shock ϵ_{2t} from Equ (2) can impact y_{1t} in a non-linear way because it can be in the domain of the function m . This construction also provides high flexibility in estimating relationships that do not take a fixed linear form, especially in cases of extreme fluctuations in time series.

To ensure statistical consistency, the model must satisfy a set of **basic assumptions**:

Assumption 1: (Weak Non-stationarity / Gao & Phillips, 2010): The error term is assumed to permit local non-stationarity, relying on the flexibility of kernel smoothers to absorb structural shocks and temporal breaks without requiring strict stationarity of the series.

$$\epsilon_{it} = \sigma_i \left(\frac{t}{n} \right) \xi_{it} \quad \dots (3)$$

$$E(\xi_{it}) = 0 \quad \dots (4)$$

$$\text{cov}(\epsilon_{1t}, \epsilon_{2t}) = \sigma_{12}^2 \quad \dots (5)$$

Assumption 2: Independence of extraneous variables.

$$E(z_{1t}, \epsilon_{1t}) = 0, E(\epsilon_{2t}/z_{1t}, z_{2t}) = 0 \quad \dots (6)$$

Assumption 3: Conditional Orthogonality / Imbens & Newey, 2009: The structural error term is orthogonal conditional on the first stage residuals, so removing the endogeneity bias using the control function approach, expressed as:

$$E(\epsilon_{1t}/z_{1t}, v_{2t}) = 0 \quad \dots (7)$$

Where v_{2t} represents the first-stage reduced-form residuals (or control function errors) obtained from cleansing the endogenous variable (y_{2t}). It acts as a conditioning proxy for the purpose of isolating and capturing the endogeneity components, thus confirming the conditional orthogonality condition needed for consistent semiparametric estimation.

Assumption 4: Homogeneity of variance

$$E(\epsilon_t \epsilon_t / z_t) = \Sigma \quad \dots (8)$$

3.1 Reduced form

To obtain the **reduced form**, we use the total differentiation method, also known as the method of simultaneous partial derivatives. This method involves differentiating both equations (1) and (2) with respect to the external variables, and then solving the resultant system to find the marginal effect of each external variable on each dependent variable.

Therefore, to begin with, the simplified form of the system that directly links dependent variables to external variables and shocks will be written as follows:

$$y_{1t} = \pi_{11} z_{1t} + \pi_{12} z_{2t} + v_{1t} \quad \dots (9)$$

$$y_{2t} = \pi_{21} z_{1t} + \pi_{22} z_{2t} + v_{2t} \quad \dots (10)$$

Where v_{1t}, v_{2t} Errors and the non-parametric function.

Now, to derive the coefficients of the internal dependent variable π_{22} which reflects the overall elasticity of response to y_{2t} the direction of shocks in z_{2t} , We derive y_{2t} from equ (2) with respect to z_{2t}

$$\frac{\partial y_{2t}}{\partial z_{2t}} = \beta_2 + \beta_3 \frac{\partial y_{1t}}{\partial z_{2t}} \quad \dots (11)$$

And the $\frac{\partial y_{1t}}{\partial z_{2t}}$ is

$$\frac{\partial y_{1t}}{\partial z_{2t}} = \alpha_3 \cdot \dot{m}(y_{2t}^*) \cdot \frac{\partial y_{2t}}{\partial z_{2t}} \quad \dots (12)$$

Substituting equation 12 into equation 11, we obtain

$$\frac{\partial y_{2t}}{\partial z_{2t}} = \beta_2 + \beta_3 \left[\alpha_3 \cdot \dot{m}(y_{2t}^*) \cdot \frac{\partial y_{2t}}{\partial z_{2t}} \right] \quad \dots (13)$$

$$\frac{\partial y_{2t}}{\partial z_{2t}} - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*) \cdot \frac{\partial y_{2t}}{\partial z_{2t}} = \beta_2 \quad \dots (14)$$

$$\frac{\partial y_{2t}}{\partial z_{2t}} [1 - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*)] = \beta_2 \quad \dots (15)$$

$$\pi_{22} = \frac{\beta_2}{[1 - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*)]} \quad \dots (16)$$

And to find the effect of the external variable z_{1t} in the equation (1) on the dependent variable y_{2t} , which represents the transitive or transient effects, i.e., to measure how shocks are transmitted from one sector to another, to measure the effect of external variables z_{1t} on the dependent variable y_{2t} , through the composite channel that passes through the first derivative of the non-parametric function $\dot{m}(\cdot)$, thus proving that the system is not discrete but structurally interconnected .we drive equ(2) with respect z_{1t}

$$\frac{\partial y_{2t}}{\partial z_{1t}} = \beta_3 \frac{\partial y_{1t}}{\partial z_{1t}} \quad \dots (17)$$

And drive equ(1) with respect z_{1t} we get :

$$\frac{\partial y_{1t}}{\partial z_{1t}} = \alpha_2 + \alpha_3 \left[\dot{m}(y_{2t}^*) \cdot \frac{\partial y_{2t}}{\partial z_{1t}} \right] \quad \dots (18)$$

Substituting equation 18 into equation 17, we obtain

$$\frac{\partial y_{2t}}{\partial z_{1t}} = \beta_3 \left[\alpha_2 + \alpha_3 \left[\dot{m}(y_{2t}^*) \cdot \frac{\partial y_{2t}}{\partial z_{1t}} \right] \right] \quad \dots (19)$$

$$\frac{\partial y_{2t}}{\partial z_{1t}} [1 - [\beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]]] = \beta_3 \alpha_2 \quad \dots (20)$$

$$\pi_{21} = \frac{\beta_3 \alpha_2}{[1 - [\beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]]]} \quad \dots (21)$$

Then, to derive the coefficients of the first internal variable π_{11} which represents the response y_{1t} to the external variables z_{1t} , we substitute (21) in equ (18) we get

$$\begin{aligned} \pi_{11} &= \alpha_2 \\ &+ \alpha_3 \left[\dot{m}(y_{2t}^*) \cdot \frac{\beta_3 \alpha_2}{[1 - [\beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]]]} \right] \quad \dots (22) \end{aligned}$$

$$\pi_{11} = \frac{\alpha_2 - \alpha_2 \beta_3 \alpha_3 [\dot{m}(y_{2t}^*)] + \alpha_2 \beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]}{1 - [\beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]]} \quad \dots (23)$$

$$\therefore \pi_{11} = \frac{\alpha_2}{1 - [\beta_3 \alpha_3 [\dot{m}(y_{2t}^*)]]} \quad \dots (24)$$

To derive π_{12} (effect of z_{1t} on y_{1t}) we substitute the equation (16) into equ(12) and obtain

$$\frac{\partial y_{1t}}{\partial z_{2t}} = \alpha_3 \cdot \dot{m}(y_{2t}^*) \cdot \frac{\beta_2}{[1 - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*)]} \quad \dots (25)$$

$$\pi_{12} = \frac{\alpha_3 \cdot \beta_2 \cdot \dot{m}(y_{2t}^*)}{[1 - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*)]} \quad \dots (26)$$

Now that the derivation process for the four marginal coefficients has been completed, the disordered form can be written in the following format.

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} \quad \dots (27)$$

$$\pi = \frac{1}{1 - \beta_3 \alpha_3 \cdot \dot{m}(y_{2t}^*)} \begin{bmatrix} \alpha_2 & \alpha_3 \cdot \beta_2 \cdot \dot{m}(y_{2t}^*) \\ \beta_3 \alpha_2 & \beta_2 \end{bmatrix} \quad \dots (28)$$

3.2 Identification of the Structural Model

To ensure the diagnostic viability of the proposed structural model, i.e., to guarantee unique and consistent estimates of structural features, especially given the presence of the non-parametric component, the diagnostic conditions established by Wooldridge (2010) regarding the Order Condition were applied to the system composed of equations (1) and (2).

First: The Order Condition: This condition is essential for diagnosis and is based on comparing the exclusions of external variables to the number of equations in the system. Applying $(K - M \geq S - 1)$ to the model:

The cornerstones of our current system are defined as follows:

S: represents the number of structural equations in the system, here $(S = 2)$.

$(S - 1)$: is the minimum required number of excluded extrins, equal to (1) .

K: represents the total number of predefined (extrinsic) variables in the system, namely z_{1t}, z_{2t}

M: represents the number of extrins included in the equation to be identified.

$(K - M)$: represents the number of extrinsic variables excluded from the equation.

In the equ (1) The external variable z_{2t} is excluded from this equation, even though it exists in the system (the second equation). Since the number of excluded

variables (1) equals the number of equations minus one ($2 - 1 = 1$), the equation is just identified.

In the equ (2): The external variable, z_{2t} is excluded from this equation, making it also satisfy the condition of degree ($1 = 1$), thus making it just identified.

Second :The Rank Condition is a sufficient condition for confirming that an equation is specific. According to this condition, the rank of the coefficient matrix for the variables excluded from the equation to be specific must be equal to (S-1). To apply this condition, we formulate the coefficient matrix of the overall system, ranking the internal variables first, followed by the external variables:

Equ	y_{1t}	y_{2t}	z_{1t}	z_{2t}	Intercept
Equ(1)	1	α_3	α_2	0	α_1
Equ(2)	β_3	1	0	β_2	β_1

According to the rule, the rank of the matrix formed from the columns of the excluded variables in the other equations must be equal to (S-1=1):

For the equation(1) :The sub-matrix is $A1 = -\beta_2$ Since z_{2t} is an external variable that influences the system $\beta_2 \neq 0$, making $|A1| \neq 0$.

Since the value of the parameter $\beta_2 \neq 0$ (a prerequisite for z_{2t} to function as a tool), the rank of the matrix is 1, which is equal to (S-1). Therefore, the first equation is specific.

For the equation(2) : The sub-matrix is $A2 = \alpha_2$ Since z_{1t} is a strong tool variable, $\alpha_2 \neq 0$, making $|A2| \neq 0$

Since $\alpha_2 \neq 0$, the rank of the matrix is 1, which is equal to (S-1). Therefore, the second equation is specific.

3.3 Estimation Methodology

This work is based on the three stage sequential estimation approach to extract the system's structural parameters. This approach is based on the multi-step semiparametric methods introduced by Rodríguez-Póo et al. (2001) which rely on a stagewise process for systematically separating the parametric components from the non-parametric structures in simultaneous equations. The non-parametric endogeneity present in the system is fixed by introducing the conditional control function approach proposed by Imbens and Newey (2009) in which the instrumental variables are replaced by first stage residual extractions. Moreover, the asymptotic properties by Akerberg et al. (2014) for multi-step semiparametric estimators are

methodologically relevant to prove the statistical efficiency and uniform convergence of this sequential, semi-informed step-by-step design. This study clearly innovates this approach by adapting these frameworks to allow for a closed form algebraic solution obtained from the reduced form, which is non-convergent with a typical iterative optimization approach.

First stage: In the first stage, the parameters of the reduced-form system are estimated. The coefficients are derived using a sequential Profile Least Squares (PLS) approach instead of using a naive and strictly linear Ordinary Least Squares (OLS) estimator that would have led to a biased and inconsistent estimation of the parameters, due to the neglected non-parametric component. The non-parametric variations are purged (concentrated) in the first stage using kernel-based local weights, following the multi-step semiparametric approach used by Rodríguez-Póo et al (2001). This means that the reduced equations are written so that the explanatory variables are orthogonal to the error term. We start with an analysis of the reduced form of the entire system, making the aggregate of all (n) observations into the structured matrices:

$$Y = Z\pi + V \quad \dots (29)$$

Where:

Y: the matrix of dependent (internal) variables with dimensions (n * G), where G is the number of equations.

Z: the matrix of explanatory (external) variables with dimensions (n * K), where K is the number of instruments including the constant.

$\hat{\pi}_i$: the matrix of unknown features to be estimated with dimensions (K * G).

V: the matrix of random errors with dimensions (n * G).

$$tr(\widehat{V}\widehat{V}) = tr(Y - Z\hat{\pi})'(Y - Z\hat{\pi}) \quad \dots (30)$$

$$tr(\widehat{V}\widehat{V}) = tr(Y'Y - Y'Z\hat{\pi} - \hat{\pi}'Z'Y + \hat{\pi}'Z'Z\hat{\pi})' \quad \dots (31)$$

$$\frac{\partial tr(\widehat{V}\widehat{V})}{\partial \hat{\pi}} = -2Z'Y + 2Z'Z\hat{\pi} \quad \dots (32)$$

By equating it to zero and multiplying the two values by $(Z'Z)^{-1}$ We get :

$$\hat{\pi} = (Z'Z)^{-1}Z'Y \quad \dots (33)$$

The method used to justify the algebraic extraction in equation (33) involves a multi-stage filtering process

that is shown to be asymptotic consistent and to uniformly converge by using the estimation principles introduced in Akerberg et al. (2014).

The second stage: In this case, the methodological difference of this study is highlighted in this part, as opposed to a study that only considers the strictly linear hypothesis, this one considers the non-linear component and its derivative through the application of a "Kernel Smoother" derived from the classical formula of Nadaraya-Watson. The purpose of this step is to mathematically complete the direct algebraic derivation. The conditional expectation function is assumed to be:

$k_h(.)$ is the normal Gaussian kernel function that is applied to dynamically weight the local observations, and h is the smoothing bandwidth parameter. The first-order derivative $\widehat{m}'(.)$ is then computed, using a stabilized finite difference approximation, from this continuous profile, while maintaining the exact algebraic consistency with the computational algorithmic execution.

$$\widehat{m}(y_{2t}) = \frac{\sum_{i=1}^n k_h(y_{2i} - y_{2t}) y_{i1}}{k_h(y_{2i} - y_{2t})} \quad \dots (34)$$

The first-order derivative $\widehat{m}(y_{2t})$ is then obtained from this continuous profile by a stabilized finite-difference operation, so that the computational algorithmic execution is algebraically consistent in order to preserve the value of the first derivative, and is defined as:

$$\widehat{m}'(y_{2t}) = \frac{\Delta \widehat{m}(y_{2t})}{\Delta y_{2t}}$$

In general, this sequential formulation is able to overcome the usual boundary bias at the endpoints of the series due to conventional Nadaraya-Watson smoothers, and therefore offers good support for the last structural link.

The third stage: Algebraic Derivation of Structural Features (α , β). This is the stage in which the reduced features are replaced by structural ones by "solving the binding constraints" of Equation (28). This method is in contrast to the iterative numerical optimization used by Gao & Phillips (2010), where direct algebraic extraction of the features avoids the numerical non-convergence problems that can arise in optimization.

From the matrix in equation (28) we obtained:

$$\hat{\beta}_3 = \frac{\hat{\pi}_{21}}{\hat{\pi}_{11}} \quad \dots (35)$$

$$\hat{\alpha}_3 = \frac{\hat{\pi}_{12}}{\hat{\pi}_{22} \hat{m}'} \quad \dots (36)$$

$$\hat{\alpha}'_2 = \hat{\pi}_{11} (1 - \hat{\beta}_3 \hat{\alpha}_3 \hat{m}') \quad \dots (37)$$

$$\hat{\beta}_2 = \hat{\pi}_{22} (1 - \hat{\beta}_3 \hat{\alpha}_3 \hat{m}') \quad \dots (38)$$

$$\hat{\alpha}_1 = \bar{y}_1 - \hat{\alpha}'_2 \bar{z}_{1t} - \hat{\alpha}_3 \hat{m}(\bar{y}_2) \quad \dots (39)$$

$$\hat{\beta}_1 = \bar{y}_2 - \hat{\beta}'_2 \bar{z}_{2t} - \hat{\beta}_3 \bar{y}_1 \quad \dots (40)$$

4. The practical Aspect and Analysis of Results

This section seeks to implement the semiparametric simultaneous equations model suggested in the theoretical part of the paper using real data on macroeconomic variables. Instead the emphasis here is on separating out the simultaneous relationship between consumption and GDP, and the nonlinearity of the behavioral relationship in consumption.

The variables were chosen based on economic theory (the income and expenditure model), and are as follows:

The Endogenous Variables:

y1: Private consumption expenditure

y2: Gross domestic product (GDP) at constant prices.

And Exogenous independent variables

Z1: Money supply (M1) in terms of constant prices (monetary policy).

Z2: Government consumption expenditure (fiscal policy).

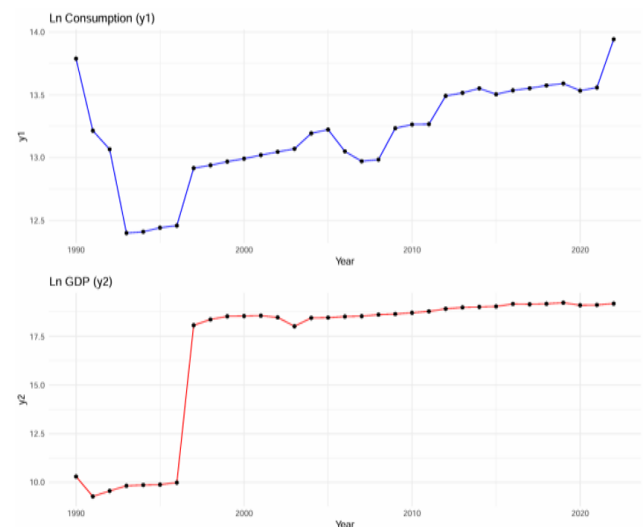


Figure (1) illustrates the trend of the private consumption and GDP variables in the period (1990-2022) with natural logarithms.

The graph shows a more abrupt structural change in both the trajectory of consumption and the GDP since 2004, indicative of a dramatic change in the structure of the economy. This is followed by a strong synchronization of their behavior and a general increasing trend in both variables over the study period. This instability and volatility of the time series supports the use of a semi-parametric elastic model that can adequately capture these structural fluctuations.

Once the timeline is reviewed, and structural breaks are identified, the strength of the interrelationships among these variables are then tested using the following correlation matrix.



Figure (2) Correlation Matrix of Study Variables

The money supply and the consumption are also very positively correlated (0.87), as well as government spending and output (0.83), which suggests the effectiveness of monetary and fiscal policies. The high values are indicative of the statistical coherence between the variables that will guarantee the correctness of the following structural estimation. In addition, the correlation between the dependent variables (0.54) shows a complex simultaneous relationship, which makes it appropriate to be used simultaneous equation methodology in this research.

Firstly the estimation was started to obtain reduced parameters that will be the basis for obtaining the structural parameters, based on the coherence of the relationships found in the correlation matrix. Using equation (33), the reduced coefficients were obtained from which the estimators $\hat{\pi}$ in equ (28) were obtained.

Table (1) Estimation Results of Reduced Form Parameters

Endogenous Variables	z1	Z2	F-statistic	p-value
y1	0.3357	0.1755	118.8	$5.579e^{-15}$
y2	-0.9645	3.6930	38.01	$5.977e^{-9}$

Results show high level of statistical significance for all the reduced parameters while the model explains about 88% and 71% of the consumption and GDP equations respectively. This provides evidence that the independent variables chosen account for the majority of the variability in the data. The values are representative of the high accuracy of the model in reduced form and serve as a solid mathematical foundation for the subsequent extraction of structural parameters.

The following figure shows the efficiency of the non-parametric part of the model in fitting the price and quantity behaviour of the variables before moving to the interpretation of the structural parameters.

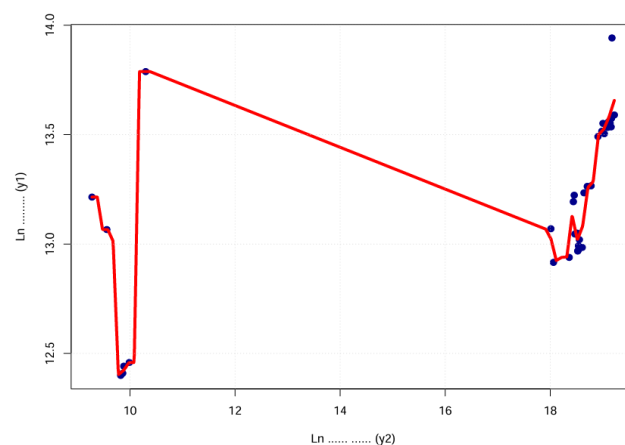


Figure (3) Estimation of Continuous Nonparametric Function for GDP and Consumption

The graph illustrates the estimation of the non-parametric behavioral function. The curve (red) accurately tracks jumps and breaks in the actual data, reflecting the non-linear relationship between GDP and consumption. The alignment of the curve with the blue dots confirms the model's success in accounting for structural changes, which enhances the accuracy of the parameter (α_3) representing the behavioral response to consumption.

To conclude the preceding steps, we arrive at the research findings, which are the structural features that reflect the net elasticities of economic policies.

Table (2) Estimation Results Structural Parameter Form Parameters

Parameters	Estimate	Standard Error (SE)	t-statistic	p-value
$\hat{\alpha}_1$	-5.878	2.0625	-2.8500	0.0078
$\hat{\alpha}_2$	0.381	0.0382	9.9823	0.0000
$\hat{\alpha}_3$	1.069	0.3101	3.4500	0.0017
$\hat{\beta}_1$	3.831	1.4191	2.7000	0.0113
$\hat{\beta}_2$	4.197	0.4573	9.1783	0.0000
$\hat{\beta}_3$	-2.872	1.0841	-2.6500	0.0127

The estimated structural parameters indicate high efficiency of both fiscal and monetary policies. Output elasticity toward government spending reached approximately 4.197, reflecting the leading role of fiscal policy. Consumption elasticity toward money supply showed a positive value of 0.381, while the non-parametric parameter of 1.069 revealed a strong behavioral response of consumption to structural shifts, confirming the model's ability to provide a more accurate analysis of net elasticities.

For validating these results econometrically, a residual test has performed to ensure the model was free from bias. and the statistical significance of all structural parameters ($p < 0.05$) confirms the validity of these elasticity estimates and confirms the existence of strong and synchronous dynamics between private consumption and economic output.

Figure (4) Residuals Analysis for Model Fit

A scatter plot of the residuals, randomly distributed about the zero line and exhibiting no geometric pattern, indicating the uniformity of variance in the model and no autocorrelation among the residuals. This random distribution is indicative of the goodness of the fit and the validity of the estimated structural features for statistical inference and the reliability in formulating recommendations.

4. Conclusions

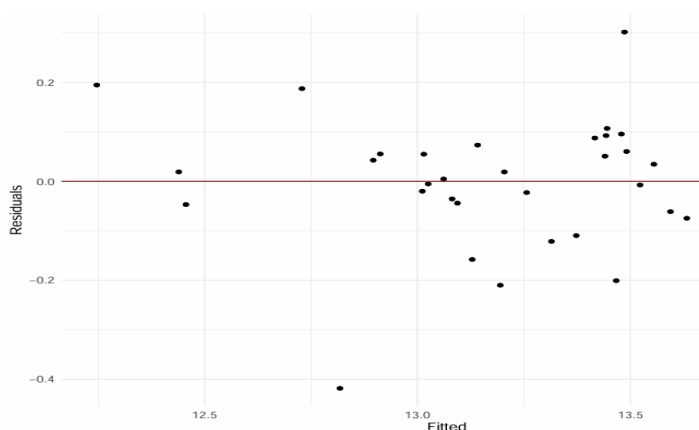
The study revealed that the sime-parametric model is effective and superior than other models in tracking the sudden structural changes that the economy of Iraq has suffered since 2004. The impact on the growth of the two policy instruments was very high in the case of the fiscal policy, as demonstrated by the structural results. Significant and positive effects of the money supply tool were also reflected in the indicators in terms of stimulating private consumption levels. The non-parametric estimate revealed a strong and non-linear behavioral reaction of consumption to the increase of GDP. The correlation matrix also verified the complexity and simultaneous relationship of the rentier economic structure based on oil revenues.

5. Recommendations

Based on the structural assessment findings, the authorities' directive to allocate spending towards capital financing to capitalize on the key effects of growth stimulus was supported, as was the modernization of credit rating channels for financial institutions to enhance the role of economic hedging in supporting income. Furthermore, adaptive stabilization rules that account for the nonlinear behavioral response of users should be implemented to mitigate the effects of structural shocks.

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