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The Quantitative Methods and Their Role in Making Administrative Decisions, The Effectiveness of Using Linear Programming in A Business Enterprise," With Reference to The Case of Libya.

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Abstract*:* Quantitative methods play an essential role in supporting administrative decision-making by offering systematic and analytical tools to evaluate alternatives and optimize resource allocation. Among these approaches, linear programming stands out as a widely used technique for solving optimization problems under specific constraints. The purpose of this study is to explore the role of quantitative methods, especially linear programming, in enhancing decision-making processes, with a particular focus on fuel distribution in Libya. Considering the operational difficulties faced by the Libyan fuel sector—such as limited transport capacity, fluctuating demand, and logistical barriers—the demand for efficient and scientifically based decision-support tools has become more urgent. To tackle this problem, a transportation model based on linear programming was developed to identify the best way to distribute fuel from multiple supply points to various demand regions while reducing total transportation costs. The model was implemented using POM-QM software, and the outcomes indicate that the proposed method greatly improves distribution efficiency and lowers operational expenses. The results of this research emphasize the practical value of quantitative methods in addressing real-world logistical problems and offer meaningful insights for strengthening resource management and decision-making within the Libyan energy sector.

***Key words*:** Operations Research, Quantitative methods, Linear Programming, Simplex Method, Libya

الأساليب الكمية ودورها في اتخاذ القرارات الإدارية، فعالية استخدام أسلوب البرمجة الخطية في مؤسسات الأعمال "مع الإشارة إلى حالة ليبيا"

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ملخص: تلعب الأساليب الكمية دورًا أساسيًا في دعم عملية اتخاذ القرارات الإدارية من خلال توفير أدوات منهجية وتحليلية لتقييم البدائل وتحسين تخصيص الموارد. ومن بين هذه الأساليب، تبرز البرمجة الخطية كإحدى التقنيات الأكثر استخدامًا في حل مشكلات الأمثلية في ظل قيود محددة. تهدف هذه الدراسة إلى استكشاف دور الأساليب الكمية، وخاصة البرمجة الخطية، في تحسين عمليات اتخاذ القرار، مع التركيز بشكل خاص على توزيع الوقود في ليبيا. ونظرًا للصعوبات التشغيلية التي يواجهها قطاع الوقود الليبي—مثل محدودية سعة النقل، وتذبذب الطلب، والعوائق اللوجستية—فقد أصبحت الحاجة إلى أدوات دعم قرار فعّالة ومبنية على أسس علمية أكثر إلحاحًا. ولمعالجة هذه المشكلة، تم تطوير نموذج نقل قائم على البرمجة الخطية لتحديد أفضل طريقة لتوزيع الوقود من عدة نقاط إمداد إلى مناطق طلب مختلفة، مع تقليل إجمالي تكاليف النقل. وقد تم تطبيق النموذج باستخدام برنامج POM-QM، وأظهرت النتائج أن الطريقة المقترحة تُسهم بشكل كبير في تحسين كفاءة التوزيع وخفض التكاليف التشغيلية. وتؤكد نتائج هذا البحث القيمة التطبيقية للأساليب الكمية في معالجة المشكلات اللوجستية الواقعية، كما تقدم رؤى مهمة لتعزيز إدارة الموارد وتحسين عملية اتخاذ القرار في قطاع الطاقة الليبي.

الكلمات الرئيسية: بحوث العمليات، الطرق الكمية، البرمجة الخطية، الطريقة البسيطة، ليبيا .

Introduction

In recent decades, quantitative methods have become essential tools in supporting administrative and managerial decision-making processes. These methods provide systematic and analytical frameworks that enable decision-makers to evaluate alternatives, optimize the use of limited resources, and address complex operational challenges in various sectors (Hillier

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& Lieberman, 2021; Panneerselvam, 2023). Their importance has grown significantly with the increasing complexity of modern organizational environments, where intuitive decision-making alone is no longer sufficient (Brandimarte, 2012; Groeneveld et al., 2015).

Among the most widely used quantitative techniques, linear programming has proven to be an effective approach for solving optimization problems involving multiple constraints (Hillier & Lieberman, 2021; Baillageon, 1996). It is particularly useful in situations that require the allocation of scarce resources in the most efficient manner, making it highly relevant in fields such as transportation, energy, agriculture, and industrial planning (Panneerselvam, 2023; Kicsiny et al., 2023).

In the Libyan context, the need for applying quantitative methods has become increasingly urgent, especially in critical sectors such as fuel distribution. The Libyan fuel supply system faces several challenges, including limited transportation capacity, fluctuating demand across regions, and logistical inefficiencies. These challenges often lead to imbalances in supply and demand, resulting in increased operational costs and reduced service reliability (El-Bouri & Ben Hmida, 2022; Abdelkader & Alsharif, 2023).

Therefore, this study focuses on the application of linear programming as a decision-support tool to improve the efficiency of fuel distribution in Libya. By developing a mathematical model that reflects real-world constraints, the study aims to determine the optimal allocation of fuel from multiple supply points to various demand locations while minimizing total transportation costs. Similar applications of linear programming have demonstrated significant improvements in operational performance and resource allocation in real-world case studies (Qarash et al., 2025; Brouer et al., 2014).

The remainder of this paper is structured as follows: the next section reviews relevant literature on quantitative methods and linear programming, followed by the methodology and model formulation. Subsequently, the results and discussion are presented, and finally, conclusions and recommendations are provided.

1. Theoretical Background

H_1: The application of linear programming leads to a reduction in fuel distribution costs in Libya.

H_2: Linear programming improves the efficiency of resource allocation in fuel distribution systems.

H_3: Changes in supply, demand, and transportation costs affect the optimal solution of the linear programming model.

2. Theoretical Background

2.1 Definition of Linear Programming and Its Assumptions

Linear programming is a mathematical optimization technique used to determine the best possible outcome, such as maximizing profit or minimizing cost, within a set of linear relationships and constraints. It is widely applied in decision-making problems where limited resources must be allocated efficiently among competing activities (Hillier & Lieberman, 2021; Panneerselvam, 2023). The technique involves the formulation of an objective function, decision variables, and a set of linear constraints that represent real-world limitations.

A linear programming model typically consists of three main components: decision variables, an objective function, and constraints. Decision variables represent the unknown quantities to be determined, while the objective function defines the goal of the problem, either maximization or minimization. The constraints, expressed as linear equations or inequalities, reflect the limitations on resources such as labor, materials, or capacity (Brandimarte, 2012; Baillageon, 1996). The effectiveness of linear programming relies on a set of underlying assumptions. One of the primary assumptions is linearity, which implies that both the objective function and the constraints must be linear in nature. This means that the contribution of each decision variable is directly proportional to its value, without any interaction or nonlinear effects (Hillier & Lieberman, 2021).

Another important assumption is divisibility, which states that decision variables can take any non-negative fractional values. This assumption is appropriate in cases where the variables represent quantities that can be divided, such as fuel or raw materials, but may not be suitable for problems involving indivisible units (Panneerselvam, 2023). The model also assumes certainty, meaning that all coefficients in the objective function and constraints are known with certainty and remain constant throughout the analysis. In real-world applications, however, this assumption may be relaxed to account for uncertainty using advanced models (Kicsiny et al., 2023).

Additionally, the assumption of non-negativity requires that all decision variables must be greater than or equal to zero, reflecting the practical reality that negative quantities of resources are not feasible. Finally, the assumption of additivity implies that the total effect of all decision variables is equal to the sum of their individual contributions, without any interaction effects (Hillier & Lieberman, 2021; Brandimarte, 2012).

These assumptions define the conditions under which linear programming models can be effectively applied. When these conditions are satisfied, linear programming provides a powerful and efficient tool for solving a wide range of optimization problems in areas such as transportation, energy distribution, and production planning (Brouer et al., 2014; El-Bouri & Ben Hmida, 2022).

2.2 Applications of Linear Programming

Linear programming has been widely applied across various sectors due to its effectiveness in optimizing resource allocation and improving decision-making processes. Its flexibility allows it to address a wide range of real-world problems where limited resources must be distributed efficiently among competing activities (Hillier & Lieberman, 2021; Panneerselvam, 2023). In the field of transportation and logistics, linear programming is commonly used to solve distribution and routing problems, aiming to minimize transportation costs while satisfying supply and demand constraints. Transportation models, in particular, are among the most important applications, as they help determine the optimal allocation of goods from multiple sources to multiple destinations (Brouer et al., 2014).

In the energy sector, linear programming plays a crucial role in optimizing the generation and distribution of energy resources. It is used to ensure efficient utilization of available capacities while minimizing operational costs and maintaining system reliability. These applications are especially important in countries facing supply constraints and infrastructure challenges (Kicsiny et al., 2023). In agriculture, linear programming is applied to maximize crop production or profit by determining the optimal combination of crops under constraints such as land availability, water resources, and labor. This is particularly relevant in arid regions where resources are scarce and must be managed efficiently (Panneerselvam, 2023).

In industrial and production planning, linear programming assists in scheduling production processes, allocating raw materials, and managing workforce utilization. It enables organizations to achieve optimal production levels while minimizing costs and meeting demand requirements (Brandimarte, 2012).

In the context of Libya, the application of linear programming is of particular importance due to the economic and logistical challenges faced by key sectors. In fuel distribution, for example, linear programming can be used to optimize the allocation of fuel from depots to various regions, taking into account transportation costs, supply limitations, and fluctuating demand. This contributes to improving efficiency and reducing operational costs (El-Bouri & Ben Hmida, 2022; Abdelkader & Alsharif, 2023).

Furthermore, empirical studies have demonstrated the effectiveness of linear programming models in improving operational performance and resource allocation in real-world scenarios, including small and medium-sized enterprises in Libya (Qarash et al., 2025).

2.3 Importance of Linear Programming in Decision-Making

Linear programming plays a fundamental role in enhancing the quality of decision-making in organizations by providing a structured and quantitative framework for analyzing complex problems. It enables decision-makers to evaluate multiple alternatives systematically and select the most efficient solution based on clearly defined objectives and constraints (Hillier & Lieberman, 2021; Panneerselvam, 2023).

One of the key advantages of linear programming is its ability to optimize the allocation of scarce resources. In many real-world situations, organizations operate under limited availability of resources such as capital, labor, and raw materials. Linear programming helps in determining the optimal combination of these resources in a way that maximizes profit or minimizes cost, thereby improving overall operational efficiency (Brandimarte, 2012).

Moreover, linear programming contributes to improving the accuracy and objectivity of decisions. Unlike intuitive or experience-based decision-making, which may be influenced by personal judgment, linear programming relies on mathematical models and quantitative data. This reduces uncertainty and enhances the reliability of the decisions made (Groeneveld et al., 2015; Esan et al., 2016). Another important aspect is its role in strategic planning and policy formulation. Linear programming models assist organizations in long-term planning by evaluating different scenarios and identifying optimal strategies under various constraints. This is particularly useful in sectors such as transportation, energy, and supply chain management, where efficient planning is critical for success (Brouer et al., 2014; Kicsiny et al., 2023).

In addition, linear programming supports decision-making in environments characterized by complexity and interdependence among variables. By considering multiple constraints simultaneously, it allows decision-makers to understand the trade-offs between different alternatives and select solutions that achieve the best overall outcome (Merigó & Yang, 2017).

In the Libyan context, the importance of linear programming is particularly evident in addressing logistical and economic challenges. For instance, in fuel distribution systems, linear programming can significantly improve the efficiency of resource allocation and reduce transportation costs by identifying optimal distribution strategies. This leads to better service delivery and enhanced operational performance (El-Bouri & Ben Hmida, 2022; Abdelkader & Alsharif, 2023).

Overall, linear programming serves as a powerful decision-support tool that enhances efficiency, reduces costs, and improves the effectiveness of managerial decisions across various sectors

3. Formulation of the General Form of the Linear Program and Solution Methods

3.1. Formulation of the General Form:

Linear programming is used to find the best allocation of limited resources and capabilities among different uses to achieve a specific objective, such as maximizing profit, production, or cost reduction, under fixed constraints and factors. The economic problem is formulated and expressed in the form of linear mathematical relationships, meaning equations of the first degree. Figure (1) below briefly illustrates the steps of modeling and solving a linear programming problem.

Figure (1) represents a summary of the decision-making process using linear programming. It begins with constructing the mathematical model of the problem based on data collected from the actual situation. This involves identifying the desired objective and defining all variables that influence it within the overall system. Then, alternative solutions are examined and studied, and systematic processes are developed to address them and achieve the desired objective. Finally, the solution is refined to reach the optimal solution.

In summary, the general form of a linear programming problem involves defining decision variables, formulating an objective function to be maximized or minimized, and specifying constraints in the form of linear inequalities or equations. The objective function and constraints are then optimized simultaneously to find the optimal solution.

A. Elements of Linear Programming Model:

The linear programming model consists of the following fundamental elements (Hillier, 2001):

- **Variables**: These are called decision variables. By determining their values, we reach the desired objective of maximizing profit or minimizing cost for the studied problem. It is required that they are non-negative and are subject to a certain type of measurement, meaning they are quantifiable. These variables are denoted by

$$X_1, X_2, X_3, \ldots\ldots\ldots\ldots\ldots, X_n$$

Where n is the number of variables in the problem.

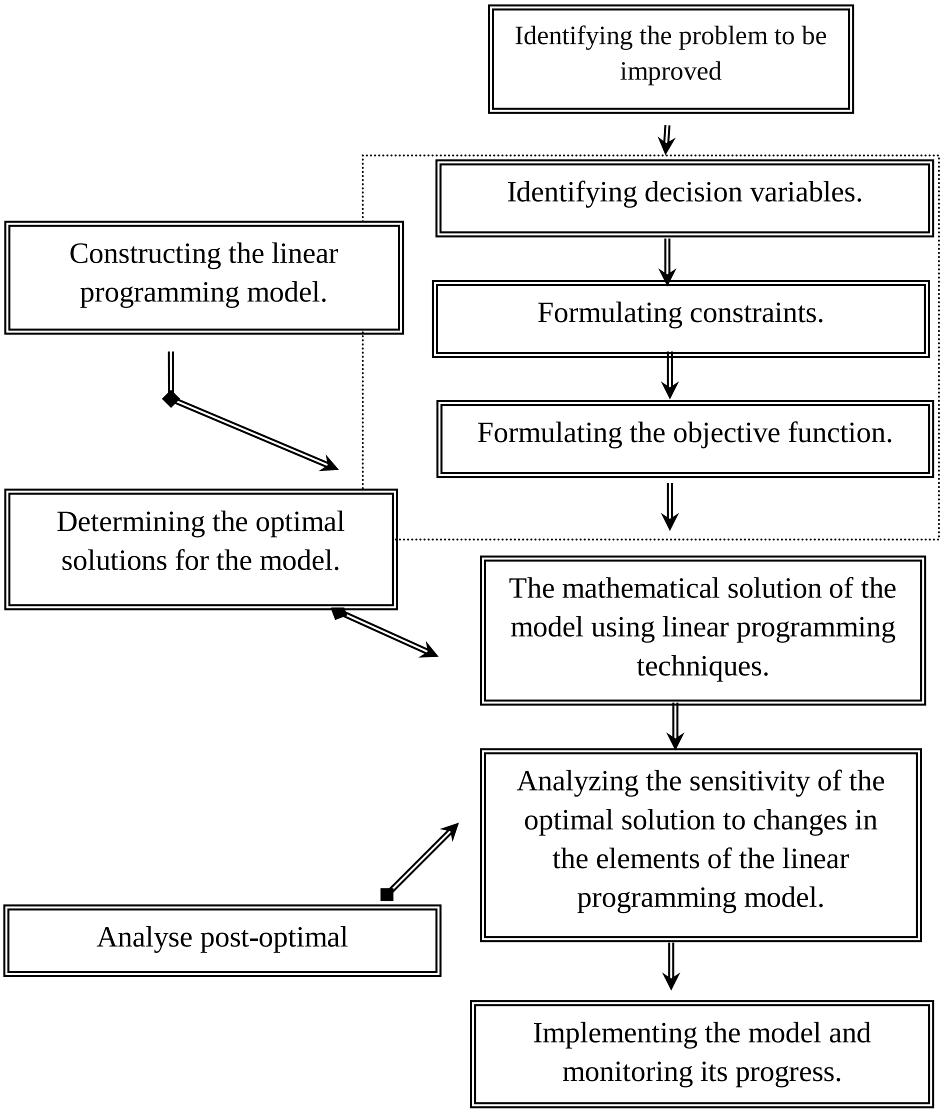


Figure (1) - Illustration of Linear Programming Modeling and Solution Steps

These variables represent one of the following concepts: quantities of production for specific products, working hours in certain departments of a factory, company, or organization, amounts of money allocated for specific activities or events, the amount of foreign currency allocated for importing certain goods, quantities of materials transported on a specific route or by specific means of transport, the quantity of raw materials needed to manufacture a specific product.

Objective Function: It is a mathematical function that represents the objective we aim to achieve, such as maximizing profit or minimizing cost. The general form of this function is (Baillageon, 1996):

$$\boldsymbol{Z = C_1X_1 + C_2X_2 + \ldots\ldots\ldots\ldots + C_nX_n} \quad (1)$$

In short،

$$\boldsymbol{Z = \sum_{J=1}^{n} C_J X_J} \quad (2)$$

Where CJ are real numbers called coefficients representing the contribution of variables in the objective function. The objectives addressed by linear programming are classified into two groups:

The first group: Maximization: This group aims to maximize the objective function, such as maximizing profit, minimizing time and effort, or maximizing national income to the greatest extent possible. The objective function will be denoted by a capital letter Z, and its goal is to maximize (MAX).

$$Max\ \boldsymbol{Z = C_1X_1 + C_2X_2 + \ldots\ldots\ldots\ldots + C_nX_n} \quad (3)$$

In short،
$$\text{Max } \boldsymbol{Z = \sum_{J=1}^{n} C_J X_J} \quad (4)$$

Where X_J represents decision variables and C_J represents unit profit for X_J.

The second group: It involves minimizing the objective function, such as aiming to reduce costs to the lowest possible level or minimizing losses as much as possible. The objective function is formulated as follows:

$$Min\ \boldsymbol{Z = C_1X_1 + C_2X_2 + \ldots\ldots\ldots\ldots + C_nX_n} \quad (5)$$

In short،
$$\text{Min } \boldsymbol{Z = \sum_{J=1}^{n} C_J X_J} \quad (6)$$

Constraints (Enghezal, 2000): represent relationships between variables, expressed mathematically as linear inequalities, taking the following forms (Simonnard, 1972):

1. First Form: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \boldsymbol{a_{ij}x_j \le b_i} \quad (7)$$

If the objective function is of the maximization type, Max.

2. Second Form: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \boldsymbol{a_{ij}x_j \ge b_i} \quad (8)$$

If the objective function is of the minimization type, Min.

The first and second forms are collectively referred to as the canonical form of the linear programming model.

3. Third form: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \boldsymbol{a_{ij}x_j = b_i} \quad (9)$$

If the objective function is of the minimization type, Min, or the maximization type, Max.

The third form, whether the objective function is maximization (MAX) or minimization (MIN), is referred to as the standard form of the linear programming model.

4. Fourth form: $$\sum_{i=1}^{m} \sum_{j=1}^{n} \boldsymbol{a_{ij}x_j} \begin{Bmatrix} = \\ \le \\ \ge \end{Bmatrix} \boldsymbol{b_i} \quad (10)$$

Whether the objective function is maximization (MAX) or minimization (MIN), is referred to as the mixed form of the linear programming model.

n: The number of variables in the linear model.

m: The number of constraints in the problem (number of linear constraints).

aij: Real numbers (coefficients).

bi: Real numbers representing available resources or requirements necessary for each constraint of the problem, and must be positive.

Non-negativity constraint: Variables must be non-negative. This condition is imposed on all models ($x_j \ge 0$) because they represent production quantities, which cannot be negative.

The general form of linear programming can be summarized using matrices to find the value of variable (X) that maximizes or minimizes the objective function (Z=CX) subject to the constraints ($AX \le B$) or ($AX \ge B$), where (C) is the coefficient matrix and ($X \ge 0$) is the constraint matrix.

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{bmatrix},\ C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_n \end{bmatrix},\ B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix},\ A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad (11)$$

The notation $X \ge 0$ means that all components of the variable X are non-negative.

3.2 Solution Methods

Several methods are used to solve linear programming problems, depending on the size and complexity of the model. The **graphical method** is one of the simplest approaches and is applicable only when the model contains two decision variables. It provides a visual representation of the feasible region and helps identify the optimal solution at one of the corner points. For larger and more complex problems, the **simplex method** is the most widely used technique. Developed by Dantzig, this method is an iterative algorithm that systematically moves from one feasible solution to another, improving the objective function value until the optimal solution is reached (Doumpos et al., 2023).

In addition, modern computational tools and software packages, such as POM-QM, are commonly used to solve linear programming problems efficiently. These tools implement advanced algorithms that can handle large-scale models and provide accurate and fast solutions, along with additional analysis such as sensitivity analysis (Hillier & Lieberman, 2021). Overall, the selection of the solution method depends on the nature of the problem, the number of variables, and the available computational resources.

3.2.1. Graphical Method

The graphical method is used to solve linear programming problems with two variables, or when a problem can be reduced to two variables. It is simple and provides a clear understanding of the feasible region and serves as a foundation for more advanced methods such as the simplex method (Kicsiny et al., 2023).

The procedure involves formulating the problem mathematically, representing constraints as straight lines on a graph, and identifying the feasible region that satisfies all constraints. The optimal solution is obtained at one of the corner points of this region. It can be determined either by evaluating the objective function at each corner point (corner point method) or by graphing the objective function and shifting parallel lines to reach the optimal point (Groeneveld et al., 2015).

3.2.2 Simplex Method

Since the graphical method is limited to problems with two variables, the simplex method is used for solving larger and more complex linear programming problems. Developed by Dantzig in 1947, it is an iterative algebraic method that systematically improves the solution until the optimal value is reached (Esan et al., 2016).

The method begins with an initial feasible solution and proceeds by evaluating the possibility of improvement. If improvement is possible, variables enter and leave the solution basis, generating a new feasible solution. This process continues until no further improvement can be achieved, at which point the optimal solution is obtained (Pesqueux et al., 1980).

3.2.3. Types of Solutions in Linear Programming

Solutions in linear programming are classified into:

- Infeasible solutions: Solutions that do not satisfy the constraints and lie outside the feasible region.
- Feasible solutions: Solutions that satisfy all constraints and non-negativity conditions, including all points within the feasible region.
- Basic feasible solutions: Corner points of the feasible region representing extreme solutions.
- Optimal solution: The best solution among feasible solutions that maximizes or minimizes the objective function (Merigó & Yang, 2017).

3.2.4. Special Cases in Linear Programming

Several special cases may arise when solving linear programming problems:

- Infeasibility: Occurs when constraints are inconsistent, resulting in no feasible solution.
- Redundancy: Arises when some constraints do not affect the feasible region.
- Unboundedness: Occurs when the feasible region is not limited, leading to no finite optimal solution, often due to missing constraints (Kicsiny et al., 2023).
- Alternate optimal solutions: Occur when multiple solutions yield the same optimal value.

4. Sensitivity Analysis and Dual Problem in Linear Programming

4.1 Sensitivity Analysis

Sensitivity analysis represents a critical tool for decision-makers, particularly in dynamic environments characterized by continuous changes in factors such as raw material prices, demand levels, and technological developments (Kicsiny et al., 2023). While classical linear programming models assume fixed parameters and complete information, real-world conditions often involve uncertainty and variability.

This approach examines the extent to which the optimal solution is affected by changes in the parameters of the model. It allows decision-makers to determine whether the current optimal solution remains valid or requires adjustment when input values change, without the need to resolve the entire model. As a result, sensitivity analysis helps reduce both time and computational effort. Specifically, it evaluates the impact of changes in the coefficients of the objective function, the availability of resources represented by the right-hand side of constraints, and the parameters within the constraints themselves. This provides valuable insights into the robustness and stability of the optimal solution under varying conditions.

4.2 The Dual Problem in Linear Programming

The concept of duality is fundamental in linear programming, as each primal problem is associated with a corresponding dual problem (de Werra, 1990). This relationship offers an alternative perspective for analyzing optimization problems and can facilitate the solution process.

The dual model can, in some cases, simplify computations and provide a more efficient path to the optimal solution. It also enables decision-makers to interpret the economic value of resources and constraints, thereby enhancing the quality of managerial decisions. When converting a primal problem into its dual form, certain conditions must be observed. For maximization problems, constraints are generally expressed in the form of "less than or equal to," whereas minimization problems involve "greater than or equal to" constraints. If variables do not conform to these conditions, they can be adjusted through appropriate transformations, such as multiplying by negative values.

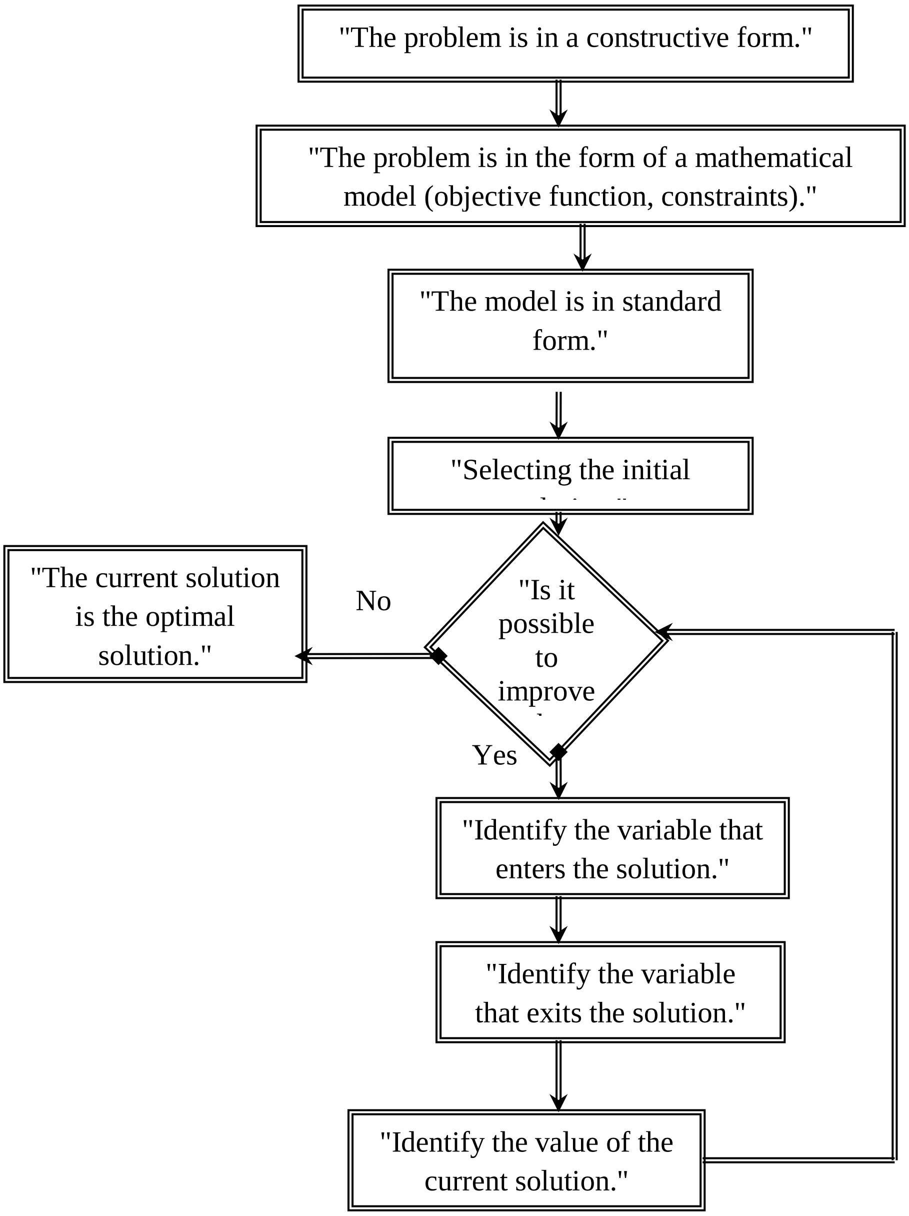
Furthermore, the solution process of linear programming models—particularly when using the simplex method—follows a sequence of structured steps aimed at improving the solution iteratively until optimality is reached. **Figure (2)** illustrates the main stages involved in applying the simplex method, including the identification of an initial solution, testing for optimality, and performing iterative improvements until the optimal solution is obtained.

5. Areas of Application of Linear Programming in Business Enterprises

Linear programming is widely used in business enterprises to optimize the allocation of limited resources among competing activities. It has proven to be an effective tool in both academic and practical contexts for addressing a variety of managerial and operational problems (Kicsiny et al., 2023). Its applications include determining the optimal allocation of production resources such as raw materials, machinery, and labor across different products in order to identify the most efficient production mix. It is also applied in planning the distribution of various types of energy according to expected future demand, as well as in designing optimal transportation plans for moving goods from multiple sources to multiple destinations, commonly referred to as transportation problems, which are central to this study.

In addition, linear programming is used to allocate resources, including personnel and equipment, to different tasks while considering variations in their capacities and performance (Mohammed, 2022). It also plays a significant role in investment decision-making by helping to distribute financial resources among alternative opportunities to achieve maximum returns.

Furthermore, linear programming contributes to financial analysis and planning by supporting decisions related to financial balance, liquidity management, cost minimization, and profit maximization through the use of financial data from actual or projected statements (Brouer et al., 2014). It is important to note that these applications represent only a subset of the potential uses of linear programming, as it is also widely applied in marketing, finance, and various decision-making contexts.



"Figure (2) illustrates the steps of solving using the simplex method."

6. Evaluation of Linear Programming Usage in Business Organizations

6.1 Criticisms of Applying Linear Programming

Despite its wide applicability, linear programming is subject to several limitations. First, it assumes linear relationships among variables, whereas real-world relationships are often nonlinear, requiring approximations that may reduce accuracy. Second, it relies on the assumption of certainty, which is rarely realistic in practice, although sensitivity analysis can partially address this limitation (Kicsiny et al., 2023).

Additionally, linear programming may not be suitable for problems involving indivisible variables, which require integer programming techniques. It may also neglect qualitative factors that cannot be expressed numerically, potentially affecting the validity of the optimal solution. Furthermore, its application often requires large amounts of data, which may be difficult to obtain, especially for small and medium-sized enterprises.

Nevertheless, linear programming remains an essential managerial tool, widely supported by computational advancements, enabling organizations to address complex decision-making problems effectively.

6.2 Difficulties in Implementing Linear Programming in Libyan Enterprises

In addition to theoretical limitations, several practical challenges hinder the implementation of linear programming in Libyan organizations (Child, 2015). One of the main challenges is the lack of reliable and sufficient data, which is essential for building accurate models. This may result from weak information systems or limited managerial capabilities in data collection and analysis.

Moreover, implementing linear programming may require restructuring administrative and accounting systems, redefining responsibilities, and ensuring the availability of qualified personnel with both mathematical and managerial expertise.

Another major difficulty lies in introducing this technology within organizations. Successful implementation requires awareness, proper planning, and training of employees. Training programs are essential not only for technical skills but also for improving communication and coordination between departments, ensuring effective application and continuity.

6.3 Advantages of Linear Programming

Despite these challenges, linear programming offers significant advantages. It enables optimal utilization of resources by considering all production factors, leading to profit maximization or cost minimization. It also improves the quality of decision-making by promoting objectivity and reliance on accurate data.

Furthermore, it contributes to enhancing managerial skills by encouraging analytical thinking and structured problem-solving. Linear programming is also flexible, allowing modifications to incorporate additional constraints or real-world considerations. Finally, it provides a comprehensive framework that integrates all production factors into a unified decision-making model (Qarash et al., 2025).

6.4 Application in the Libyan Context

The fuel sector in Libya represents a critical area where linear programming can play a vital role due to its connection to transportation, electricity, and public services. Challenges such as limited transport capacity, fluctuating demand, and multiple supply sources highlight the need for optimization models.

Linear programming contributes to improving resource allocation and operational efficiency across key sectors, including agriculture, energy, transportation, and industry. Empirical studies support its effectiveness in Libya. For instance, Qarash et al. (2025) demonstrated its ability to enhance profitability in industrial settings, while El-Bouri & Ben Hmida (2022) showed its effectiveness in reducing transportation costs. Similarly, Abdelkader & Alsharif (2023) confirmed its role in optimizing fuel distribution and improving supply efficiency under operational constraints.

7. Advanced Applied Study: A Linear Programming Model for Fuel Distribution in Libya: Problem Description: This research aims to improve the efficiency of fuel distribution in Libya by developing a linear programming model that considers multiple supply sources (depots) and multiple demand centers (cities), while striving to reduce the overall transportation cost under the constraints of supply, demand, and distribution capacity.

Network Structure Model: The system consists of, 3 Depots (Sources): Tripoli, Misrata, Benghazi. 4 Cities (Destinations): Tripoli, Zawiya, Sirte and Benghazi. This represents the Transportation Network Problem model.

Cost matrix		Destinations				
Sources	From \ To	Tripoli	Zawiya	Sirte	Benghazi	Supply
	Tripoli	2	3	6	10	120
	Misrata	5	4	3	6	100
	Benghazi	10	8	4	2	80
	Demand	90	70	60	80	

The table above represents the basic data used in building the linear programming model for fuel distribution, which is divided into three main elements: supply, demand, and transportation costs.

The supply values indicate the available quantities of fuel at each depot. The table shows that the Tripoli depot provides 120,000 liters per day, Misrata 100,000 liters, and Benghazi 80,000 liters. These values represent the maximum quantities that can be shipped from each source. The demand values indicate the fuel needs of each city. Tripoli needs 90,000 liters per day, Zawiya 70,000 liters, Sirte 60,000 liters, and Benghazi 80,000 liters. These values represent the quantities that must be fully supplied to ensure system equilibrium.

The transportation cost matrix represents the cost per unit of fuel when transported from each depot to each city, measured in cost units (such as dinars per thousand liters). These values reflect the differences in distances or transportation conditions; the cost is lower the closer the depot is to the city, and vice versa. This data thus forms the essential input for the mathematical model, which the POM-QM program relies on to find the optimal solution for reducing overall cost while balancing supply and demand. Solution using POM-QM, the model was solved using POM-QM for Windows (Transportation Module) with the Vogel Approximation + MODI Optimization method.

$$Min\ Z = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij}$$

$$Subject\ to:$$

$$\sum_{j}^{4} x_{ij} = a_i, \qquad i = 1,2,3,4$$

$$\sum_{i}^{3} x_{ij} = b_j, \qquad j = 1,2,3$$

$$x_{ij} \geq 0$$

Total transportation cost = 770 cost units. The results show that: using geographically closest depots significantly reduces costs. The most influential depot is Tripoli due to its strategic location. A perfect balance between supply and demand was achieved without shortage or surplus.

Sensitivity Analysis

1. Change in Demand: When demand in Sirte increases by 10%: demand rises to 66 units. This leads to redistribution from Misrata, resulting in a slight increase in total cost (~3–5%).

2. Change in Transportation Costs: If the transportation cost from Misrata → Sirte increases from 3 to 5: reliance on this route decreases, part of the supply is diverted from Tripoli, increasing total cost by approximately 6–8%.

Conclusion: Based on sensitivity analysis, the model is robust to minor changes but relatively sensitive to transportation costs on intermediate routes. The results confirm that linear programming is an effective tool for improving fuel distribution efficiency in Libya, as the model contributed to reducing operational costs, improving resource allocation, and supporting logistical decision-making. The model also highlights the importance of integrating quantitative analysis into oil supply chain management in constrained environments such as Libya.

Optimal Distribution Table:

Transportation Results

(untitled) Solution

solution value = $770	Tripoli	Zawiya	Sirt	Benghazi
Tripoli	90	30		
Misrata		40	60	
Benghazi			0	80

Final Solution Table - Shipments and Marginal Costs

(untitled) Solution

	Tripoli	Zawiya	Sirt	Benghazi
Tripoli	90	30	[4]	[10]
Misrata	[2]	40	60	[5]
Benghazi	[6]	[3]	0	80

Sensitivity Analysis

1. Change in Demand: When demand in Sirte increases by 10%: demand rises to 66 units. This leads to redistribution from Misrata, resulting in a slight increase in total cost (~3–5%).

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This application demonstrates that using a linear programming transportation model via POM-QM software can significantly improve fuel distribution efficiency, making it a suitable tool for application in Libyan oil companies.

8. Conclusion

This study aimed to evaluate the effectiveness of linear programming in optimizing fuel distribution in Libya through the development of a transportation model solved using POM-QM software.

The results demonstrated that the proposed model successfully achieved an optimal allocation of fuel between depots and demand centers while fully satisfying supply and demand constraints. The optimal total transportation cost was found to be 770 cost units, indicating a high level of efficiency in minimizing distribution costs.

The findings also revealed that geographical proximity plays a significant role in cost reduction, as depots located closer to demand centers—particularly Tripoli and Misrata—were more intensively utilized in the optimal solution. Moreover, the model ensured a perfect balance between supply and demand, with no shortages or surpluses observed.

Sensitivity analysis showed that the model is relatively stable under minor changes in demand, but more sensitive to variations in transportation costs along key routes. These changes led to measurable adjustments in distribution patterns and total cost, confirming the responsiveness of the model to operational variations.

In relation to the research hypotheses, the results confirmed that linear programming contributes to reducing fuel distribution costs, improving resource allocation efficiency, and responding effectively to changes in key parameters. This highlights its value as a practical decision-support tool in the Libyan fuel sector.

Overall, the study confirms that linear programming provides an effective and reliable approach for improving fuel distribution systems, supporting cost reduction, and enhancing supply chain efficiency in constrained environments such as Libya.

9. Recommendations

Based on the findings of this study, the following recommendations are proposed to enhance fuel distribution efficiency and support decision-making in Libya:

Adoption of Linear Programming Models: Decision-makers in the fuel sector should adopt linear programming as a standard tool for planning and resource allocation, given its proven ability to reduce transportation costs and improve distribution efficiency.

Improvement of Data Systems: Institutions should develop integrated and reliable data collection systems for fuel supply, demand, and transportation costs, as accurate data significantly enhances the effectiveness of optimization models.

Implementation of Decision-Support Software: The use of optimization software such as POM-QM should be encouraged, as it facilitates solving complex distribution problems and improves the quality and speed of decision-making.

Capacity Building and Training: Training programs should be provided for managerial and technical staff in operations research and quantitative methods to ensure effective application of linear programming techniques.

Enhancement of Transportation Infrastructure: Since transportation costs significantly influence the optimal solution, improving logistics networks and transportation efficiency should be prioritized to achieve further cost reductions.

Future Research Directions: Future studies are encouraged to develop more advanced models incorporating uncertainty, dynamic demand, and real-world constraints specific to Libya, such as economic and operational challenges.

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