Sirte University Scientific Journal Vol. 15, No. 1 (2025) 67-76

Sirte University Scientific Journal (SUSJ)

Journal home page: http://journal.su.edu.ly/index.php/SUSJ/index

DOI: 10.37375/susj.v15i1.3388

Time effects on deflection of reinforced concrete beams with large web opening

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DOI: 10.37375/susj.v15i1.3388

ARTICLE INFO:

Received 08 April 2025.
Accepted 10 May 2025.
Available online 24 June2025

Keywords: beam (reinforced concrete), opening, long-term deflection, concrete creep, concrete shrinkage, design structure.

A B S T R A C T

The objective of the present study is to elucidate the influence of creep and shrinkage on the long-term deflection of reinforced concrete beams that contain a large opening in the web under service sustained loads while also providing practical equations for the design of deflection in reinforced concrete structures predicated on linear creep theory. To achieve this, the simplified equations used in the present EC2, ACI code, and ACI435 are rigorously evaluated and contrasted. In addition, 4RC simply supported reinforced concrete beams (3RC beams with large web opening and 1RC beam without opening) under sustained mid-span load were made and tested, to validate the proposed method. The proposed method, based on a new simplified curvature coefficient, is found simple to use, amenable to programming on compact calculators, and exhibits a high degree of concordance with empirical results, thus, it is concluded that the proposed equations possess a suitable level of practicality for adoption in practice.

1 Introduction

In contemporary building construction, the network of pipes, electrical cables, and ventilation ducts is provided in the roof or ceiling space. The incorporation of these conduits through apertures in the floor beams eliminates a significant amount of dead space and results in a more compact and economical design, especially in multistory buildings.

However, including an opening in the web of an RC beam leads to many issues in the beam behavior, such as:

a-reduction in the beam stiffness.

b- excessive cracking.

c-excessive deflection significantly in the area of web openings by decreasing the moment of inertia at the opening.

d- change the beam force behavior to an extra complex one.

Most of the research focused on the problem of the strength of RC beams containing large web openings. In contrast, relatively scant research has been devoted to the analysis of satisfactory serviceability performance, especially regarding the deflection behavior of such beams under service loads.

Important research dealing with the problem of short-term deflection in reinforced and prestressed concrete beams with web opening has been published by Barney et al., 1977, Mansur et al., 1992, Mansur 2006, Alves and Scanlon 1985, Mansur and Kiang-Hwee 1999, and Pertiwi et al., 2021.

Until recently, very little testing had been undertaken on the long-term deflection of RC beams with web openings under sustained service loads.

This research endeavors to elucidate the ramifications of creep and shrinkage on the long-term deflection of RC beams that contain a large opening in the web under sustained service loads and provide a practical equation for the design of deflection in RC structures based on linear creep theory.

2 Short-term deflection

The model of the beam with an opening is illustrated in Fig. (1). The initial configuration of the beam, characterized by a rectangular opening, is depicted in Fig.(1-a). The chord members are situated above and below the opening. As demonstrated in Fig.(1-c), under the assumption that the ultimate failure occurs by a mechanism formation comprising four hinges in the chords, one at each corner of the opening.

The short-term (instantaneous) deflection due to opening can be obtained as shown in Fig.(1-d):

$$f_o(t_0) = \alpha . l \quad (1)$$

And

$$\theta = \frac{\alpha . l}{L} \quad (2)$$

$$\alpha = \frac{f_o(t_0)}{l} \quad (3)$$

Substituting α of Eq. 3 in Eq. 2 gives:

$$\theta = \frac{f_o(t_0).l}{L.l} \quad (4)$$

$$f_o(t_0) = \theta . L \quad (5)$$

the rotation at each end of the opening can be calculated as:

$$\theta = \frac{q.l^3}{12.E.I.L} \quad (6)$$

substituting (θ) of Eq. (6) in Eq. (5) gives the deflection at opening (typically occurs near the end of the opening high-moment):

$$f_o(t_0) = \frac{q.l^3}{12.E.I} \quad (7)$$

Therefore, mid-span deflection of beam due to opening:

$$f_1(t_0) = k . f_o(t_0) \quad (8)$$

Calculated the sum of the deflection in Eq. (8) and the deflection obtained for the beam without an opening $f_2(t_0)$, gives the total short-time deflection of the beam:

$$f(t_0) = f_1(t_0) + f_2(t_0) \quad (9)$$

In which:

q = shear force at the opening, in the case of concentrated load we have $q = p/2$.

l = opening length, E= modulus of elasticity of concrete.

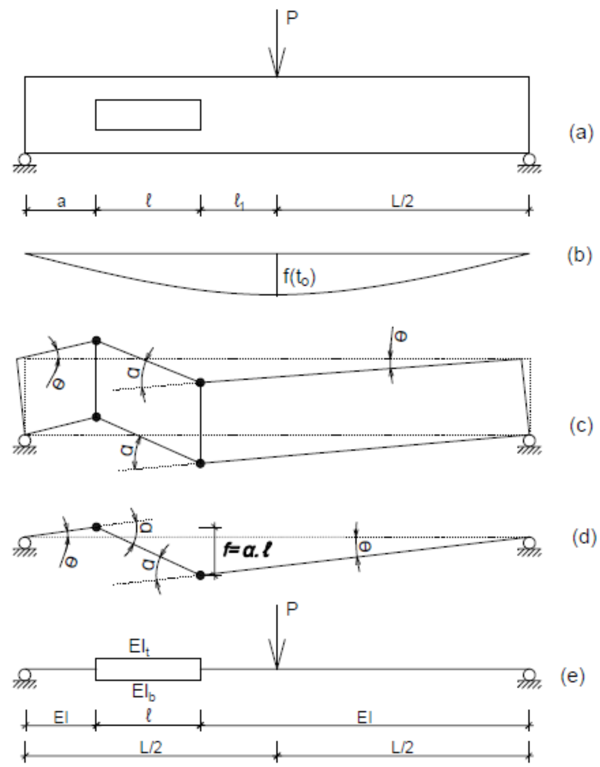

Figure 1. Deflection due to web opening: (a) beam with opening, (b) deflection of the beam without opening, (c) deformed shape of the beam (d) differential deflection across the opening (e) EI (rigidity) along the length of the beam.

$I = I_{t+}I_b$ moment of inertia for the chords.

θ= the curvature (angle of rotation at ends).

$$k = \frac{L}{L+2l_1} \text{ (for mid-span deflection).} \quad (10)$$

l_1= distance from high moment end of opening to mid span beam Fig. (1).

3 Comparison with measured results

A comparison is made between the maximum instantaneous deflection under sustained load computed using the proposed method and the measured deflection reported by (Mansur et al., 1992), for simply supported beams, each of which contains a rectangular opening. The loading and details of these beams are summarized in Fig. (2) and Table 1.

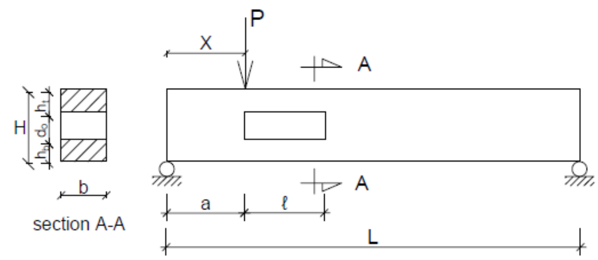

Table 2 shows that the calculated deflections correlate well with the measured deflections and (Mansur et al., 1992). Generally, but not entirely, the procedure proposed in this paper seems to slightly overestimate the deflection in the case of the simple beam (R_{14}) with a (large) opening L=1m, one explanation could be that the effective length of the opening is long relative to the span of the beam.

The ratios of measured to calculated deflection using the proposed method (Eq. 9) range from 0.84 to 1.04,

Figure 2. simply supported RC beams.

beam	x_{mm}	l_{mm}	$h_{t\ mm}$	$d_{0\ mm}$	$h_{b\ mm}$	a_{mm}
R_1	1000	400	110	180	110	1800
R_2	1000	600	110	180	110	1700
R_3	1000	800	110	180	110	1600
R_6	1000	800	130	140	130	1600
R_7	1000	800	90	220	90	1600
R_{11}	1000	800	110	180	110	1600
R_{14}	1500	1000	110	180	110	1750
L= 3m for all beams						

Table 1- beam Details tested by Tan (1982) (Mansur et al., 1992).

Beam	Load kN	Measured deflection mm	Proposed Method mm	Mansour Method mm	$\frac{measured}{proposed}$	$\frac{measured}{Mansour}$
R_1	131.7	6.62	6.35	6.40	1.04	1.03
R_2	107.4	6.70	6.60	6.64	1.01	1.01
R_3	84.8	7.11	7.19	7.04	0.99	1.01
R_6	105.9	7.26	7.59	6.77	0.96	1.07
R_7	60.5	6.79	6.92	8.71	0.98	0.78
R_{11}	78.5	6.87	7.016	6.64	0.98	1.03
R_{14}	44.7	8.27	9.80	7.82	0.84	1.06

Table 2- Comparison between proposed method, Mansour method (Mansur et al., 1992) and measured instantaneous deflections (Mansur et al., 1992).

″All beams were rectangular in cross-section, 200mm wide, and 400mm deep. Eight bars arranged symmetrically across the section and continuing throughout the length. Provided the longitudinal reinforcement in each beam. Further details of these tests can be found in the relevant reference (Mansur et al., 1992) ‟.

with an average of 0.97 and a coefficient of variation of 3.5 per cent.

Similarly, using the (Mansur et al., 1992) method, the ratios range from 0.78 to 1.07, with an average of 0.99 and a coefficient of variation of 0.8 per cent.

4 Long-term deflection

The increments in the curvature $\Delta\varphi$ caused by creep and shrinkage define the change in deflection during the period t_0 to t.

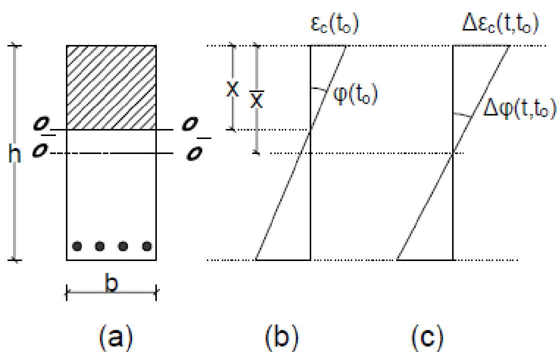

Figure 3. $\mathcal{E}_c(t_0)$, $\Delta\mathcal{E}_c(t_0,t)$, $\varphi(t_0)$ and $\Delta\varphi((t_0,t)$ in a fully cracked RC section.

$\Delta\varphi$=$\Delta\varphi_{cr}$+$\Delta\varphi_{sh}$
(11)

For sustained loads, the total curvature including creep $\varphi_{cr}(t)$, can be determined by using the

age - adjusted effective modulus of elasticity of concrete (AEMM), defined as (Bazant, 1972):

$\overline{E_C}(t,t_0) = \frac{E_C(t_0)}{1+\chi\theta(t,t_0)}$
(12)

Where:

$E_C(t_0)$= modulus of elasticity of concrete.

χ= aging coefficient of concrete.

$\theta(t,t_0)$= creep coefficient of concrete.

According to Eq. (12) the age - adjusted effective modulus of elasticity $\overline{E_C}(t,t_0)$ decreases due to creep at a rate of $(1+\chi\theta(t,t_0))$. As a result, to reach equilibrium, the depth of the compression zone and curvature of a cracked section subjected to a constant bending moment both increases with time Fig. (3).

the total curvature $\varphi_{cr}(t)$ due to creep effect, is equal to:

$$\varphi_{cr}(t)=\Delta\varphi_{cr}(t,t_0)+\varphi(t_0) \quad (13)$$

The additional (increment) curvature due to creep can be defined as:

$$\frac{\Delta\varphi_{cr}(t,t_0)}{\varphi(t_0)} = \frac{\varphi_{cr}(t)}{\varphi(t_0)} - 1 \quad (14)$$

Where:

$\Delta\varphi_{cr}(t,t_0)$ = the increment curvature during the period (t,t_0) due to creep.

$\varphi(t_0)$ =the curvature at time (t_0).

Assuming that the section is subjected to a constant M, therefore, the ratio $\frac{\varphi_{cr}(t)}{\varphi(t_0)}$ is equal to ratio $\frac{E(t_0).I}{\bar{E}(t,t_0).\bar{I}}$ as shown in the following equation:

$$\frac{\varphi_{cr}(t)}{\varphi(t_0)} = \frac{\frac{M}{\bar{E}(t,t_0).\bar{I}}}{\frac{M}{E(t_0).I}} = \frac{E(t_0).I}{\bar{E}(t,t_0).\bar{I}} \quad (15)$$

$$\frac{\varphi_{cr}(t)}{\varphi(t_0)} = \frac{E(t_0).I}{\frac{E(t_0)}{1+\chi\theta(t,t_0)}.\bar{I}} \quad (16)$$

$$\frac{\varphi_{cr}(t)}{\varphi(t_0)} = \left(1+\chi\theta(t,t_0)\right)\frac{I}{\bar{I}} \quad (17)$$

Using Eqs. (14) and (17), the relative increment of curvature during the period (t,t_0) can be written as:

$$\frac{\Delta\varphi_{cr}(t,t_0)}{\varphi(t_0)} = \left(1+\chi\theta(t,t_0)\right)\frac{I}{\bar{I}} - 1 \quad (18)$$

$\Delta\varphi_{cr}(t,t_0) = \gamma_{cr}.\varphi(t_0)$
(19)

$\gamma_{cr} = (1+\chi\theta(t,t_0))\frac{I}{\bar{I}} - 1$
(20)

Where γ_{cr} is the proposed simplified curvature coefficient depending on the geometrical properties of section, and the product $\chi.\theta(t,t_0)$.

The curvature due to shrinkage $\Delta\varphi_{sh}$ can be determined as (CEN 2004, 2004; Saker, 2007; Saker, 2006; Saker, 1999):

$$\Delta\varphi_{sh} = -\varepsilon_{sh}(t, t_0).\frac{\overline{S_b}}{\bar{I}} \tag{21}$$

$$\Delta\varphi_{sh} = -\varepsilon_{sh}(t, t_0).\bar{\omega}_{sh} \tag{22}$$

$$\bar{\omega}_{sh} = \frac{\overline{S_b}}{\bar{I}} = \frac{A_C.y_c}{\bar{I}} \tag{23}$$

Where:

$\varepsilon_{sh}(t, t_0)$ is the free shrinkage during the period (t, t_0).

A_C is the cross-section area of concrete.

y_c is the y-coordinate of the centroid of A_C .

$\bar{I}$ is the moment of inertia of the (AEMM) transformed section about its centroid.

Therefore, the total long-term deflection in simply supported beams with opening, $f(t)$, is given as the sum of instantaneous, $f(t_0)$, and the time-dependent deflection, $f(t, t_0)$:

$$f(t) = f(t_0) + f(t, t_0) \tag{24}$$

Where the time-dependent deflections is the sum of creep deflection, $f_{cr}(t, t_0)$, and shrinkage deflection $f_{sh}(t, t_0)$:

$$f(t, t_0) = f_{cr}(t, t_0) + f_{sh}(t, t_0) \tag{25}$$

$$f(t, t_0) = \gamma_{cr}.f(t_0) + \alpha.\varepsilon_{sh}(t, t_0).\bar{\omega}_{sh}.L^2 \tag{26}$$

Where α depends on the support conditions of the beam ($\alpha = 0.125$ for simple beam).

5 Comparison with experimental data

A simply supported 3RC beam with a large web opening and 1RC beam without an opening subjected to a sustained load (P) at mid-span, were made and tested to validate the proposed method.

The relationships between the measured results and the calculated results using the proposed method are shown in figures (4,5,6 and 7).

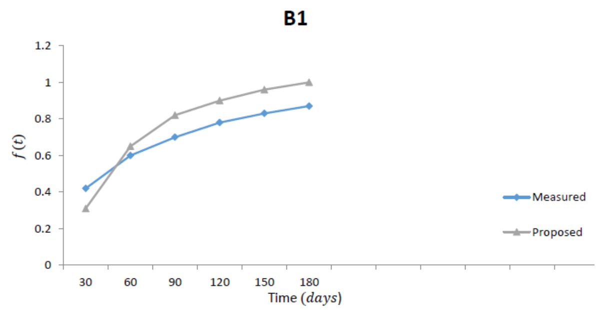

Figure 4. Comparison of the proposed method and measured total mid-span deflection for B_1.

Beam	f_c (28)N/mm^2	x_{mm}	l_{mm}	$h_{t\ mm}$	$d_{0\ mm}$	$h_{b\ mm}$	a_{mm}	*Load(kN)*
B_1	21.6	750	300	80	60	60	310	1.2
B_2	21.3	750	-	-	-	-	-	2.3
B_3	21.3	750	280	70	65	65	320	2.3
B_4	31	750	300	70	65	65	300	1.6+0.6*kN/m*
L=1.5m, H=200mm, B=100mm for all beams. B_2 without opening (solid beam).								

Table 3- Details of tested beams.

The beams under consideration were all 100 mm in width and 200 mm in depth. The length of each beam was precisely 1500mm. As illustrated in Fig. (2), the effective length *L*=1430mm, and the tensile reinforcement consisted of two bars of 8mm diameter. The longitudinal reinforcement in the top chord (bot. and top steel) and bottom chord (top steel) was 2Ø6mm, the shear-span was reinforced with steel stirrups (6mm diameter/200mm), and the clear cover was 15mm. The modulus of elasticity *E* of the concrete is taken according to the ACI building code (ACI 318, 2008).

The loading, details and obta318ined results of these beams are summarized in Tables 3 and 4.

As illustrated in Table 4, the calculated total deflections using the proposed method Eq. (26), agrees well with the measured deflections for all tested beams.

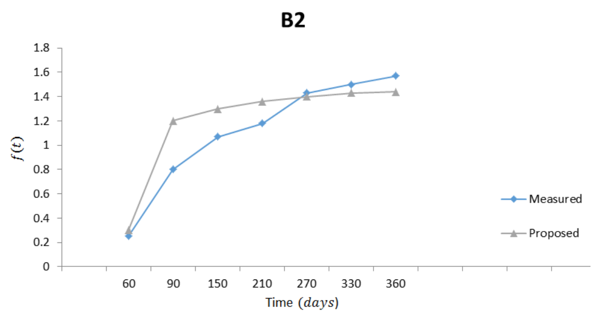

Figure 5. Comparison of the proposed method and measured total mid-span deflection for B_2.

Beam	*t_0 days*	*$f(t_0)$ mm*		*measured / proposed (1)/ (2)*	*f(t) mm*		*measured / proposed (3)/ (4)*
		Measured deflection (1) mm	*Proposed Method (2) mm*		*Measured deflection (3) mm*	*Proposed Method (4) mm*	
B_1	30	0.42	0.31	1.35	0,87	1.00	0.87
B_2	60	0.25	0.30	0.83	1.57	1.44	1.09
B_3	30	0.50	0.45	1.10	1.68	1.62	1.03
B_4	30	0.41	0.38	1.08	0.92	1.10	0.84
B_2 without opening (solid beam), L=1.5 for all beams.							

Table 4- Comparison between proposed method and measured deflection at mid-span beams.

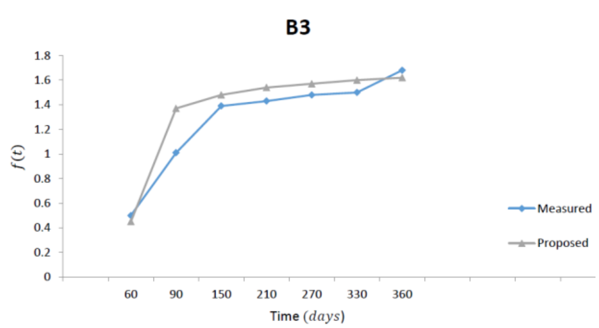

Figure 6. Comparison of the proposed method and measured total mid-span deflection for B_3.

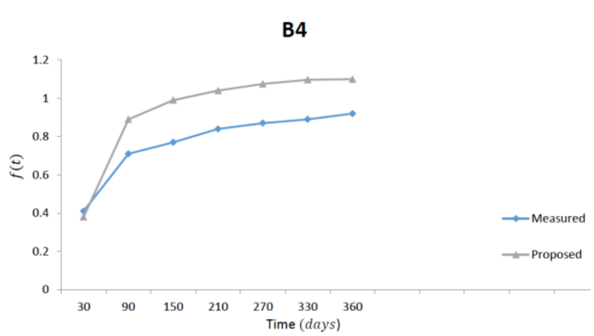

Figure 7. Comparison of the proposed method and measured total mid-span deflection for B_4.

Furthermore, as illustrated in Table 4, for a total of four beams, the ratio of measured to calculated instantaneous

deflections using the proposed method $\frac{f(t_0)\boldsymbol{m}}{f(t_0)\boldsymbol{c}}$ range from 0.83 to 1.35, with an average of 1.09, and a coefficient of variation of 8.7 per cent. Similarly, for total deflections $\frac{f(t)\boldsymbol{m}}{f(t)\boldsymbol{c}}$, the ratio ranges from 0.84 to 1.09 with an average of 0.96 and a coefficient of variation of 2.3 per cent.

6 Comparison with different methods

The objective of this comparison is to investigate the error in the long-term deflection when using the ACI 435-209 (ACI Committee 435, 1978; ACI Committee 435, 1995; ACI committee 209, 1971), ACI Code method (ACI 318, 2008), CEB-MC 90-99 **(**CEB-FIP 1990), and the proposed method with the simplified coefficient $\boldsymbol{\gamma_{cr}}$. A comparison is carried out using a simply supported beam with a rectangular section subjected to sustained load at mid-span beam *R14* (Table 1). The values of free shrinkage strain $\boldsymbol{\varepsilon_{sh}}$, and of creep coefficient $\boldsymbol{\theta(t,t_0)}$, have been considered according to the ACI 209 model, a typical value of 0.8 has been taken for the aging coefficient of concrete $\boldsymbol{\chi(t,t_0)}$ used in the formulation of CEB-FIP.

The graphs in Fig. (8) and (9) show the Variation of γ_{cr}, k_r, k_{cr}, and k_{sh}, $\overline{\omega}_{sh}$, k_{sh} coefficients versus time since loading, using different methods.

It is important to note that the equation provided by the ACI code Eq.(A1) yields a single λ value which depends only on $\rho^{'}$ and the time dependent parameter ζ (equivalent to 1 to 2). It is evident that a direct comparison between λ and proposed simplified coefficient γ_{cr} is not possible. The value of k_{cr} , k_r and γ_{cr} are plotted and compared in Fig. (8).

In addition, Fig. (8) shows that, the k_r-value calculated by the ACI 209 and ACI 435R codes Eq.(A7) would be above the range of the graphs. On the other hand, the k_{sh}-value calculated by Eq.(A5) would be below the range of the graphs in Fig. (9). Consequently, the long-term deflection cannot be accurately predicted using coefficients derived from the empirical equations Eq. (A1, A5, A6 and A7). For these reasons, it is proposed here that the ACI method does not provide a suitable equation for predicting creep and shrinkage deflections.

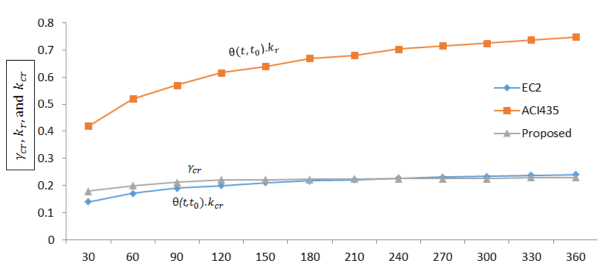

Figure 8. Variation of γ_{cr}, k_r, and k_{cr} with time.

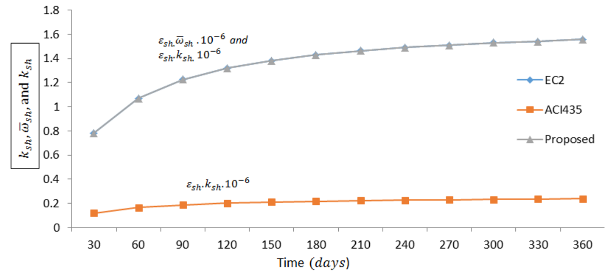

Figure 9. Variation of k_{sh}, $\overline{\omega}_{sh}$, and k_{sh} with time (notice that the proposed method and EC2 graphs are congruent) .

As we can see from results, the CEB manual coefficient k_{cr} Eq.(A16), and the proposed coefficient γ_{cr} Eq. (20), show the same trend with small differences (less than 10%).

Therefore, it can be concluded that the total mid-span deflection for the ACI 209-435 and ACI method, overestimates the total mid-span deflection as shown in Fig. (10,11,12 and 13). Consequently, any general expression for calculation of deflection of RC beams due to creep and shrinkage of concrete should take into account the following factors: steel rates, sectional properties, age of concrete, value of concrete creep coefficient, and shrinkage strain.

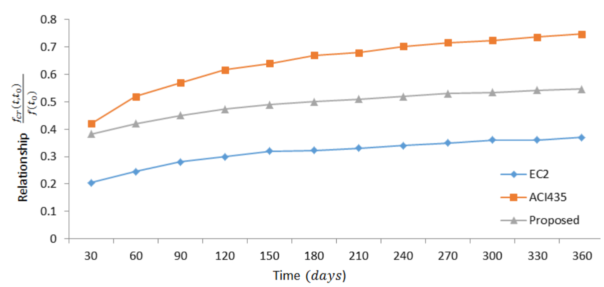

Figure 10. Relationships between the long-term deflection due to creep and instantaneous deflection.

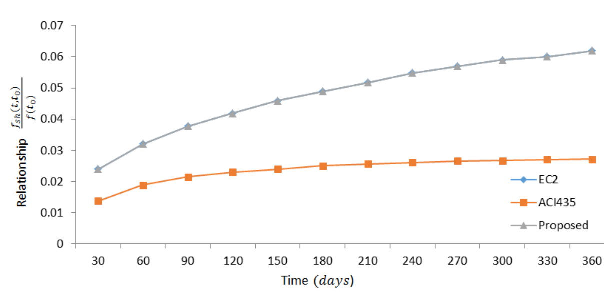

Figure 11. Relationships between the long-term deflection due to shrinkage and instantaneous deflection (notice that the proposed method and EC2 graphs are congruent) .

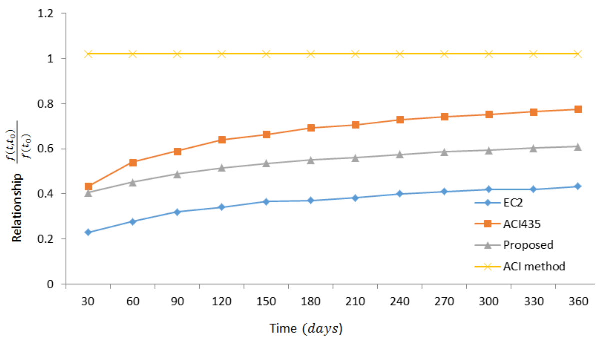

Figure 12. Relationships between the long-term deflection and instantaneous deflection.

However, the proposed method Eq. (26), and the CEB manual (CEB-FIP, 1990) give similar predictions for all cases.

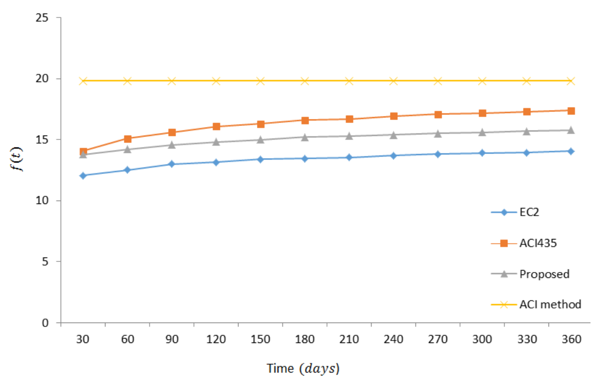

Figure 13. Comparison of $f(t)$ using different methods.

7 Conclusions

A novel, streamlined approach for assessing time-dependent deflections of RC beams with web openings has been put forward. The method has been deduced from general principles based on the Age-Adjusted Effective Modulus Method (AEMM), from an analytical development of the equations for time-dependent curvature. A novel simplified coefficient has been proposed to predict time-dependent deflections due to the creep of concrete beams, with or without web openings. The proposed coefficient is characterized by simplicity, accuracy, and the implicit inclusion of effects on sectional properties, age of loading, the creep coefficient value, and the shrinkage strain value.

In order to validate the proposed method, four simply supported RC beams (3RC beams with large web opening and 1RC beam without opening) under sustained mid-span load were made and tested. The experimental results indicate that the calculated deflection using the proposed method correlates well with the measured deflection.

Finally, a comparison using different methods is made to investigate the error in the long-term deflection when using the ACI 435-209, ACI Code method, CEB-MC 90-99, and the proposed method with the simplified coefficient γ_{cr}. The investigation revealed that the CEB-MC coefficient and the proposed coefficient γ_{cr} analogous trends, with small differences (less than 10%) for all cases.

It has also been shown that using the ACI method is not suitable for the calculation of deflections of RC beams due to creep and shrinkage of concrete.

Conflict of interest:

The author declare that there are no conflicts of interest.

APPENDIX. Long-term deflection according to the ACI and EC2

The equations presented below are based on the requirements of the Eurocode 2, ACI code, and ACI committee 435.

1-ACI code method:

According to ACI code method (ACI 318, 2008), additional long-term deflection, $f(t,t_0)$, resulting from creep and shrinkage of flexural members, is obtained by:

$$f(t,t_0) = \lambda \, . \, f(t_0)$$
(A1)

Where:

$$\lambda = \frac{\zeta}{1+50\,\rho'}$$

$f(t_0)$ is the instantaneous deflection caused by the sustained load, $\rho' = \frac{A_S'}{b.d}$ is the compression reinforcement ratio on the critical section, A_S' is the area of compression reinforcement, b is the width of compression face of member, d is the distance from extreme compression fiber to the centroid of tension reinforcement, and ζ is a time-dependent factor equal to 2.0, 1.4, 1.2, 1.0, respectively for 5 years or more, 12 months, 6 months, and 3 months.

The total deflection of the beam, $f(t)$, is given by

$$f(t) = f(t_0) + f(t, t_0) \quad \text{(A2)}$$

2-ACI 435-95 method:

A procedure for computing the deflection due to creep and shrinkage separately was recommended by ACI committee 435 (ACI Committee 435, 1978), based on the work of (Branson, 1963), as modified improvement in the prediction of creep and shrinkage (ACI committee 209, 1971).

Thus

$$f(t, t_0) = f_{cr}(t, t_0) + f_{sh}(t, t_0) \quad \text{(A3)}$$

$$f(t, t_0) = k_r . \theta(t, t_0) . f(t_0) + \alpha . \phi_{sh} . L^2 \quad \text{(A4)}$$

$$\phi_{sh} = 0.7 \frac{\varepsilon_{sh}}{h} \left(\rho - \rho^{'}\right)^{\frac{1}{3}} . \left(\frac{\rho - \rho^{'}}{\rho^{'}}\right)^{\frac{1}{2}} \text{ for } \left(\rho - \rho^{'}\right) \leq 3\% \quad \text{(A5)}$$

and

$$\phi_{sh} = \frac{\varepsilon_{sh}}{h} \quad \text{for } \left(\rho - \rho^{'}\right) > 3\% \quad \text{(A6)}$$

$$k_r = \frac{0.85}{1 + 50\rho^{'}} \quad \text{(A7)}$$

$$\theta(t, t_0) = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right) . \theta_u \quad \text{(A8)}$$

Where α= is a factor relating to the geometry of the support system, L = span length of the beam, ϕ_{sh} =shrinkage curvature, $\theta(t, t_0)$ =creep coefficinent, θ_u =ultimate creep coefficient, recommended average is 2.35 for 40% humidity, t=time in days after loading, k_r =Compression steel factor, and $f(t_0)$ =instantanuos deflection due to sustained load.

3-Eurocode 2 method:

The EC2 analysis (CEN 2004, Eurocode 2, 2004), of long-term curvature is based on an interpolation formula relating two states: state I, uncracked, and state II. The mean curvature is expressed as:

$$\psi = (1 - \zeta)\psi_{\mathrm{I}} + \zeta . \psi_{\mathrm{II}} \quad \text{(A9)}$$

$$\zeta = 1 - 0.5\left(\frac{M_{cr}}{M}\right)^2 \quad \text{(A10)}$$

where the subscripts I and II refer to uncracked and fully cracked conditions respectively, ζ is an interpolation coefficient to account for tension stiffening, M_{cr} is the cracking moment, M is the applied moment.

The long-term curvature will increase due to the effects of creep and shrinkage of concrete. For sustained loads, the total curvature including creep, $\psi(t, t_0)$, can be calculated by using the effective modulus of elasticity for concrete according to the following expression:

$$E_c(t, t_0) = \frac{E_c(t_0)}{1 + \theta(t, t_0)} \quad \text{(A11)}$$

Where $E_c(t_0)$ is the elastic modulus of concrete at time t_0 and $\theta(t, t_0)$ is the creep coeffienet that depends on time and duration of loading.

The total deflections, $f(t)$, can be obtained by interpolating between state I and state II (Ghali A, & Favre R., Concrete Structures: stresses and deformations,1994):

$$f(t) = 0.5\left(\frac{M_{cr}}{M}\right)^2 . f_I(t, t_0) + \left(1 - 0.5\left(\frac{M_{cr}}{M}\right)^2\right) . f_{II}(t, t_0) \quad \text{(A12)}$$

Where, the predicted total deflections for each state I and II can be obtained as the sum of the initial deflection, $f(t_0)$, the long-term deflection due to creep, $f_{cr}(t, t_0)$, and the long-term deflection due to shrinkage, $f_{sh}(t, t_0)$:

$$f(t)i = f(t_0)i + f_{cr}(t, t_0)i + f_{sh}(t, t_0)i \quad \text{(A13)}$$

Where subscript i takes the value of state I and II.

The long-term deflections due to creep and shrinkage are obtained separately from the following expressions:

$$f_{cr}(t, t_0)i = k_{cr.i} . \theta(t, t_0) . f(t_0)i \quad \text{(A14)}$$

$$f_{sh}(t, t_0)i = \frac{\varepsilon_{sh}}{d} . \frac{l^2}{8d} k_{sh.i} \quad \text{(A15)}$$

Being $k_{cr.i}$ and $k_{sh.i}$ the curvature coefficients related to creep and shrinkage, respectively:

$$k_{cr.i} = \frac{I_{c,i}(t, t_0) + A_{c,i} y_{c,i}(t, t_0) \Delta y(t, t_0)}{\bar{I_\iota}(t, t_0)} \quad \text{(A16)}$$

$$k_{sh.i} = \frac{A_{c,i} y_{c,i}(t,t_0) d}{\overline{I_i}(t,t_0)} \quad \text{(A17)}$$

Where $A_{c,i}$ is the area of concrete considered effective (entire concrete area in state I, but only area of compression zone in state II), $I_{c,i}(t,t_0)$ is the moment of inertia of $A_{c,i}$ about an axis through the centroid of the age-adjusted transformed section, $y_{c,i}(t,t_0)$ is the centroid of $A_{c,i}$ of measured downwards from the centroid of the age-adjusted transformed section, $\Delta y(t,t_0)$ is the y-coordinate of the centroid of the age-adjusted transformed section, measured downwards from the centroid of the transformed section at t_0, and $\overline{I_i}(t,t_0)$ is the moment of inertia of the age-adjusted transformed section about an axis through its centroid.

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