المجلة العلمية لكلية التربية – المجلد الثالث– العدد الأول – يناير 2024

Faculty of Education journal – Volume 3 Issue: 1 – January 2024

الموقع الالكتروني للمجلة: http://journal.su.edu.ly/index.php/edujournalj/index

DOI: https://doi.org/10.37375/foej.v3i1.2587

Deformation Parameters and Electric Intrinsic Quadrupole Moment for Even- Even Cerium $_{58}$Ce Isotopes

A.E Ezwam
Tripoli University Tripoli-Libya
eayad1@yahoo.com

S.A. Sallam
Tripoli University Tripoli-Libya
sa.sallam@uot.edu.ly

الملخص:

ركزنا في هذا العمل على دراسة اشكال النوى الزوجية – الزوجية لنظائر $^{124-150}_{58}\mathrm{Ce}$. ودراسة معامل التشوه β المشتق من احتمال الانتقال الكهربائي المنخفض للتحولات الكهربائية $B(E2)\uparrow$ للانتقالات $0^+ \rightarrow 2^+$ ، وتم حساب معامل التشويه δ من عزم رباعي الاقطاب الكهربائي الذاتي Q_0. العلاقة بين معلمتي التشوه والعدد السحري النيوتروني $N = 82$. تم دراسة العدد من خلال رسم معاملات التشوه كدالة لعدد النيوترونات. أظهرت النتائج أن معاملات التشوه وعزم رباعي الاقطاب الذاتي Q_0 للنواة تتناقص عندما يقترب عدد النيوترونات من العدد السحري. في العمل الحالي تمت دراسة علاقة أخرى هي الحصول على النسبة δ / β .تم في هذا البحث دراسة معدل الجذر التربيعي لنصف القطر النووي $\langle r^2 \rangle^{1/2}$ والاحداثيان الكبير والصغير للقطع الناقص a و b الذي يمثل شكل التشوه النووي بالإضافة الى الفرق بينهما.

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Tripoli University Tripoli-Libya
eayad1@yahoo.com

S.A. Sallam
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Abstract

In this work we focused on the studying of even – even nuclei forms for $^{124-150}_{58}\text{Ce}$ .The study of deformation parameter β derived from the reduced electric transition probability $B(E2)\uparrow$ for $0^+ \rightarrow 2^+$ transitions, and distortion parameter δ is calculated from intrinsic electric quadrupole moment Q_0.The relationship between two deformation parametersand β, δ . neutron magic number $N = 82$ was studied through plotting the deformation parameters β, δ as a function of neutrons number. The results show that the deformation parameters and intrinsic electric quadrupole moment Q_0 of nucleus decreased when the neutrons number approaches to magic number. In the present work, another relationship was studied; this was getting the ratio of δ / β .

In this paper also the major a and the minorellipsoid axis were calculated, and the b different between them have been determined.

Key words: Distortion parameter, Intrinsic electric quadrupole moment, reduced electric transition probability, Mean squared charge distribution radius.

INTRODUCTION

The nucleus takes on a spherical shape when the number of nucleons is equal to the magic number, and it will be more stable (Doornenbal,2013). The intrinsic electric quadrupole moment Q_0 is zero for a nucleus that has a spherically symmetric charge distribution. Deformation occurs only when both the neutron and proton shells are partially filled (Jone,2011) and the shape of the nucleus can be determined by the electric quadrupole moment Q_0, which is a measure of the deviation of the nuclear charge distribution from spherical symmetry. The study of the deformations of the nuclei is important to understand their shapes that multiply when the intrinsic electric quadrupole moment is positive and flatten when the intrinsic electric quadrupole moment is negative.

THEORY

The deformed nuclei are found in the mass ranges $150 < A < 190$ and $A > 220$ the deformation parameter β is related to the intrinsic electric quadrupole Q_0 moment by the formula (Ertugral et al.,2015;Margraf et al.,1993):

$$\beta = \frac{\sqrt{5\pi}}{3}\frac{Q_0}{ZR_0^2} \tag{1}$$

Where:

R_0: Nuclear charge radii which calculated from $R_0 = r_0 A^{1/3}$ and $r_0 = 1.2\,\mathrm{fm}$ (Hammen,2013).When the deformation parameter is positive $\beta > 0$,the nucleus takes the elongated shape from the condensed ellipsoid shape, and when it is negative $\beta < 0$,the nucleus has the flat shape from the oblate oval shape, and it is zero $\beta = 0$ for the nucleus that has a spherically symmetric charge distribution (Watkins,2011), as in figure (1).

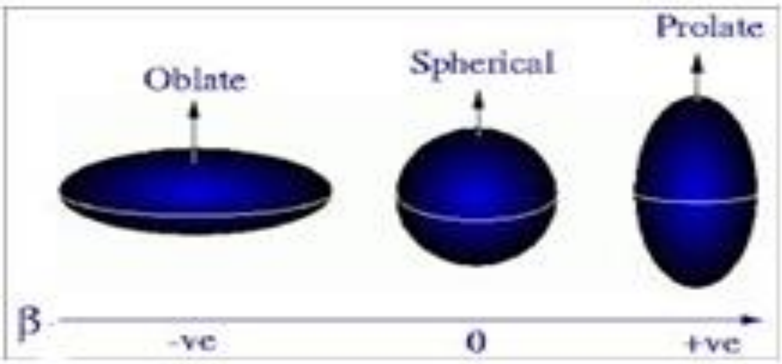

Figure (1) A diagrammatic representation of three a nuclear shapes oblate, spherical and prolate respectively. The arrows for the oblate and prolate shapes indicate the symmetry axis.

The deformation for deformed nuclei is linked to the intrinsic electric quadrupole moment of the nucleus (Regan,2003), which is in turn related to the reduced transition probability (Abdulkaadhim,2016):

$$Q_0 = \frac{4}{e}\left[\frac{\pi B(E2)\uparrow}{5}\right]^{1/2} \tag{2}$$

Where:
e: denotes the electric charge of proton.
$B(E2)\uparrow$ is the reduced electric quadrupole transition probability where the transition from the ground 0^+ state to the first excited 2^+ state.
Where the upward-pointing arrow represents excitation.
The low electric quadrupole transition probability is calculated from the following equation (Raman et al.,2001):

$$B(E2)\uparrow = 2.6E^{-1}Z^2A^{-2/3}(\mathrm{e}^2\mathrm{b}^2) \quad (3)$$

Where:
E is the energy of Gamma ray transition in KeV units, of the first excited state 2^+ was obtained from.
Z: is the atomic number, and A: is the mass number of a nucleus.
Deformation parameter β also related to the reduced electric quadrupole transition probability $B(E2)\uparrow$ by the following formal (Boboshin,2007; Ridha,2009):

$$\beta = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E_2)}{e^2}\right]^{1/2} \quad (4)$$

Where:

$$R_0^2 = \left(1.2A^{1/3}\ \mathrm{fm}\right)^2 = 0.0144A^{2/3}(\mathrm{b})$$

The distortion parameter δ can be calculated using the following equation (Boboshin,2007; Krane,1988):

$$\delta = \frac{0.75Q_0}{Z\left\langle r^2\right\rangle} \quad (5)$$

Where:
$\left\langle r^2\right\rangle$: the mean squared charged distribution radius, which is calculated by the following equation when $A > 100$ as (Jarallah and Hassan,2016):

$$\left\langle r^2\right\rangle = 0.6R_0^2 \quad (6)$$

By the two following equations can be determine the major a and the minor b ellipsoid axis as (Boboshin,2007; Ridha,2009):

$$a = \sqrt{\left\langle r^2\right\rangle\left(1.66 - \frac{2\delta}{0.9}\right)} \quad (7)$$

$$b = \sqrt{5\left\langle r^2\right\rangle - 2a^2} \quad (8)$$

While the difference between the major a and the minor b of ellipsoid axis can be calculated by three methods as (Boboshin,2007; Jarallah and Hassan,2016; Roy and Nigam,1967):

$$\Delta R_1 = \delta R_0, \Delta R_2 = b - a, \Delta R_3 = \frac{\beta}{1.06}R_0$$

CALCULATIONS AND RESULTS

In this work, the deformation parameters and intrinsic electric quadrupole moment of the nucleus Q_0 have been calculated to calculate the distortion parameter δ .However the nucleus deformation parameter β is obtained from the reduced electric quadrupole transition probability $B(E2)\uparrow$ for ground state 0^+ to the first excited 2^+ state. The mean squared charged distribution radius $\langle r^2 \rangle$ also calculated to know the oblate or prolate shapes by the major and the minor ellipsoid axis. All the above parameters have been calculated for even-even nuclei of cerium isotopes. We noticed $Z=58$ which have an atomic number $^{124-150}Ce$ that the values of the deformation modulus and the intrinsic electric quadruple moment of the nucleus, were falling back from the lower values at the neutron magic number. Where the intrinsic quadruple electric moment was plotted as a function of the neutron numbers as in the figure (2). This figure shows that the values of the intrinsic electric quadrupole moment decreased with the increase in the numbers of neutrons until reaching the magic number of neutrons, and then the electric quadrupole moment increases with the increase in the numbers of neutrons, which indicates the deviation of the charge distribution from the spherical shape. The figure also shows the variability of the calculated results with the experimental results for Q_0. At neutron magic number $N=82$ values of deformation parameters $\beta=0.0839$ $\delta=0.0793$ and $Q_0=1.429$ which represent less values of these parameters in this element as shown in table (1).

In figure (2) the experimental data of $Q_0(\text{exp})$ and our calculation $Q_0(\text{calc})$ of the intrinsic electric quadrupole moments are plotted together as a function to neutrons number. This figure shows that the both values of $Q_0(\text{calc})$ and $Q_0(\text{exp})$ are decreased with increasing of neutrons number, until reach to magic number of neutron $N=82$ closed shell of neutrons, which has low values of intrinsic quadrupole moment. However, the $Q_0(\text{exp})$ and $Q_0(\text{calc})$ are increases with increasing the neutrons number until the ratio of them approaches one.

Table (1) The quadrupole intrinsic electric Q_0, deformation factors β, δ and the ratio between them δ / β for ${}^{124-150}_{58}\mathrm{Ce}$.

No	Nucl	A	N	Experimental Data				Present Work			
				$E(2^+)$ [KeV]	Q_0 [b]	$B(E2)$	β	Q_0 [b]	β	δ	δ / β
1	$_{58}$Ce	124	66	142.0	6.100	3.70	0.385	4.990	0.317	0.300	0.946
2		126	68	169.6	5.170	2.68	0.325	4.542	0.286	0.270	0.946
3		128	70	207.3	4.780	2.28	0.298	4.087	0.255	0.241	0.946
4		130	72	254.0	4.180	1.74	0.258	3.673	0.226	0.214	0.946
5		132	74	325.5	4.330	1.87	0.265	3.228	0.197	0.186	0.946
6		134	76	409.1	3.230	1.04	0.195	2.865	0.173	0.164	0.946
7		136	78	552.2	2.850	0.81	0.170	2.454	0.147	0.139	0.946
8		138	80	788.7	2.130	0.45	0.126	2.043	0.121	0.115	0.946
9		140	82	1596.2	1.731	0.30	0.102	1.429	0.084	0.079	0.946
10		142	84	641.3	2.197	0.48	0.128	2.244	0.130	0.123	0.946
11		144	86	397.4	2.880	0.83	0.166	2.838	0.163	0.155	0.946
12		146	88	258.5	3.380	1.14	0.193	3.503	0.200	0.189	0.946
13		148	90	158.5	4.430	1.96	0.251	4.453	0.252	0.238	0.946
14		150	92	97.1	5.700	3.30	0.320	5.663	0.317	0.300	0.946

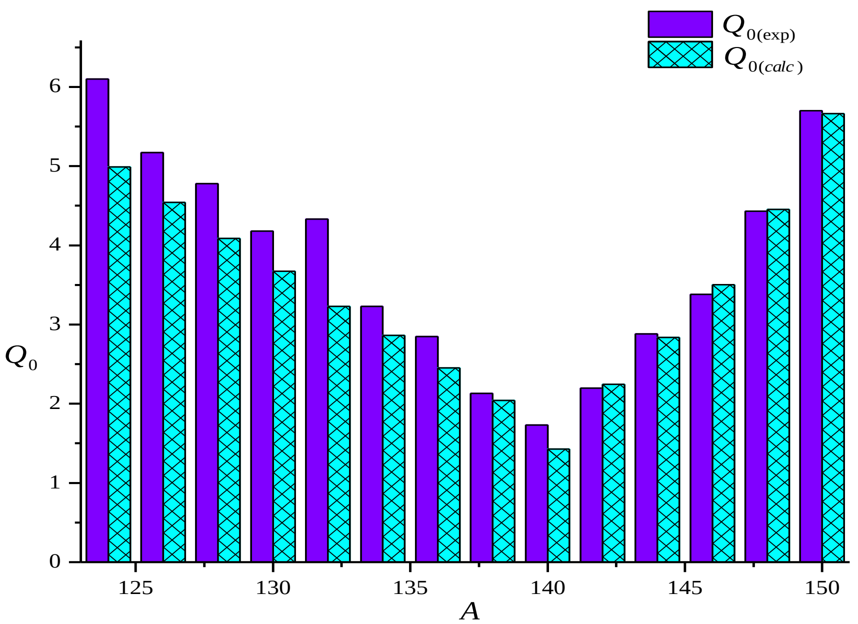

Figure (2) The experimental and calculation values of quadrupole moments as a function to neutrons number N

In figure (3) we can see the deformation parameter β for $^{124-150}_{58}\mathrm{Ce}$ represent less values at the closed shell (neutron magic number) $N=82$ and by increasing neutron numbers the deformation parameters also increase.

Also ,we can show clearly from figure (3) that the values of β are larger than δ for all our results, which means that deformation which comes from $B(E2)\uparrow$ is larger than the deformation which comes from Q_0 . Because the values of β are affected by dynamic nucleus deformation.

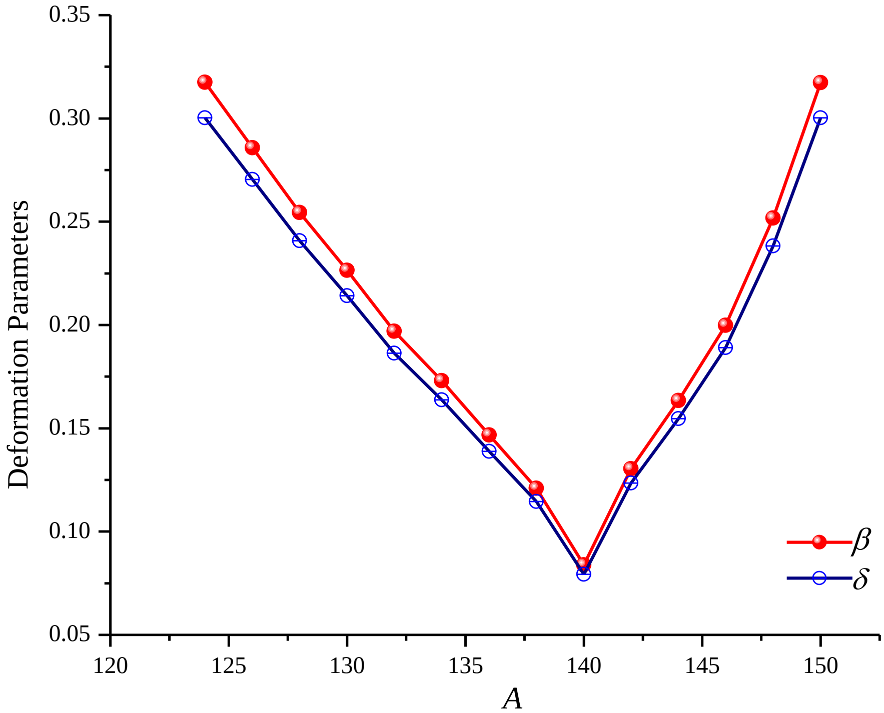

Figure (3). Comparison between β and δ values as a function to neutrons number for $^{124-150}_{58}\mathrm{Ce}$ isotopes.

We also calculated the difference between the main and secondary axes in three different ways, and the results were very close, and the table (2) shows the decrease of the difference up to the magic number $N=82$ which has been increasing, which confirms that the nucleus at the neutron magic number is a spherical nucleus.

Table (2) root mean square radii $\langle r^2 \rangle^{1/2}$,major and minor axis a,b and the difference between them for ${}^{124-150}_{58}\mathrm{Ce}$.

No	Nucl	A	N	Exp.Data		Present Work				
				R_0	$\langle r^2 \rangle$	a [fm]	b [fm]	ΔR_1	ΔR_2	ΔR_3
1	$_{58}$Ce	124	66	5.984	0.215	7.081	5.351	1.730	1.797	1.797
2		126	68	6.016	0.217	7.017	5.447	1.571	1.627	1.627
3		128	70	6.048	0.219	6.951	5.541	1.410	1.456	1.456
4		130	72	6.079	0.222	6.893	5.628	1.264	1.302	1.302
5		132	74	6.110	0.224	6.827	5.718	1.109	1.139	1.138
6		134	76	6.141	0.226	6.778	5.796	0.982	1.005	1.005
7		136	78	6.171	0.228	6.718	5.878	0.840	0.857	0.857
8		138	80	6.201	0.231	6.658	5.960	0.698	0.710	0.710
9		140	82	6.231	0.233	6.552	6.064	0.488	0.494	0.494
10		142	84	6.261	0.235	6.756	5.997	0.759	0.773	0.772
11		144	86	6.290	0.237	6.908	5.957	0.951	0.972	0.972
12		146	88	6.319	0.240	7.071	5.907	1.163	1.195	1.194
13		148	90	6.347	0.242	7.286	5.822	1.464	1.512	1.512
14		150	92	6.376	0.244	7.545	5.702	1.843	1.914	1.914

CONCLUSIONS

The deformation of the nucleus can be studied by calculating the deformation coefficient β dependent on the probability of a low electric transition $B(E2)\uparrow$ from the ground spin 0^+ state to the first excited spin 2^+ state . Determined the distortion parameter δ which depends on intrinsic electric quadrupole moment Q_0.The ratio of δ/β approaches one when the number of neutrons increases. Reducing the values of the deformation parameters and the intrinsic electric quadrupole moment is close to the magic number which means that the shape of nuclei with magic numbers of neutrons having a closed shell encourages spherical shape.

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